Geometric Mapping of Finite Elements on Shell Geometry

Project Report : Numerical Modeling of Failure Processes in Materials

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Contents

Nomenclature

$\mathbf{E^3}$: euclidean 3-space of the model

Ψ : parametric space of the finite element ($\mathbf{R^2}$)

$\mathbf{U}$: parametric space of the geometric model ($\mathbf{R^2}$)

$\mathbf{X}$: position vector in $\mathbf{E^3}$, $\mathbf{X} = \{x_1, x_2, x_3\}$

ξ : position vector in Ψ, $\xi = \{\xi_1, \xi_2, \xi_3\}$

u : position vector in $\mathbf{U}$, $u = \{u_2, u_2\}$

1 Introduction

The geometric mapping of a finite element describes the relationship between the finite element parametric space, Ψ, and the euclidean space of the model, $\mathbf{E^3}$. The mapping determines the position vector, $\mathbf{X}$, and derivatives of $\mathbf{X}$ with respect to ξ upto the required order. The accuracy of the finite element solution depends upon the accuracy of the geometric mapping of the finite elements. This dependency is more significant in the p-version of the finite element method where the finite element meshes are typically coarse [1].

The standard technique for mapping finite elements involves approximate fitting techniques which are improved iteratively based on some user specified error criterion. This report describes a different approach for mapping of the finite elements by utilizing the underlying parametric representation of the true geometric model.

2 Geometric Mapping

The problem of geometric mapping between the finite element space, Ψ, and the euclidean space of the model can be mathematically represented as

$$\mathbf{X} = \mathbf{X}(\xi) = \{x_1(\xi_1, \xi_2, \xi_3), x_2(\xi_1, \xi_2, \xi_3), x_3(\xi_1, \xi_2, \xi_3)\} \tag{1}$$

for a triangular elements using the barycentric coordinates and as

$$\mathbf{X} = \mathbf{X}(\xi) = \{x_1(\xi_1, \xi_2), x_2(\xi_1, \xi_2), x_3(\xi_1, \xi_2)\} \tag{2}$$

for a quadrilateral element. Figure shows the mapping from the Ψ space to the $\mathbf{E^3}$ space.

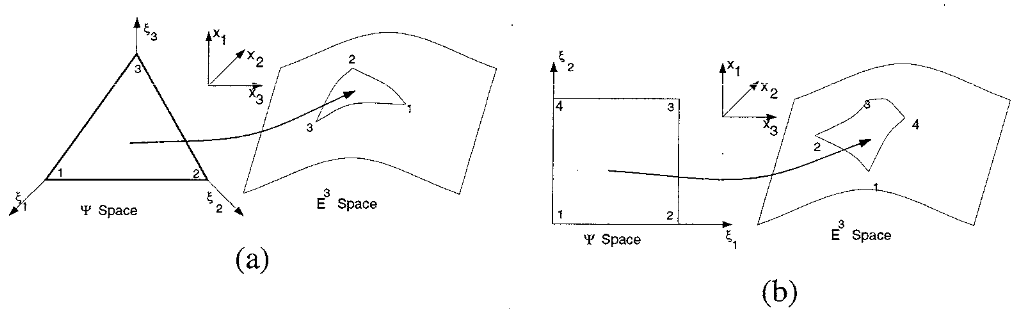

Figure 1 Mapping of a finite element face (a) Triangular (b) Quadrilateral

The approach presented here utilizes the exact geometric representation available from the modeler by constructing an intermediate mapping between the finite element space, Ψ, and the parametric space of the geometric model, $\mathbf{U}$. The geometric mapping between the $\mathbf{U}$ and the $\mathbf{E^3}$ spaces can be written as$\mathbf{X} = \mathbf{X}(\mathbf{u}) = \{x_1(u_1, u_2), x_2(u_1, u_2), x_3(u_1, u_2)\}$Figure shows the parametric representation of geometry within the geometric modeler.

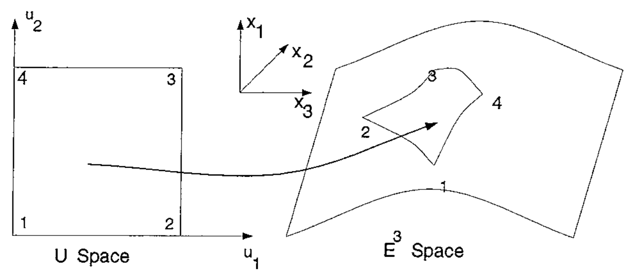

Figure 2 Mapping within the Geometric Modeler

The final mapping between the Ψ space and $\mathbf{E^3}$ space becomes a two step procedure. The first step maps the Ψ space to the $\mathbf{U}$ space and the next step utilizes the mapping of $\mathbf{U}$ to $\mathbf{E^3}$ available in the modeler. The overall process is schematically depicted in figure 3.

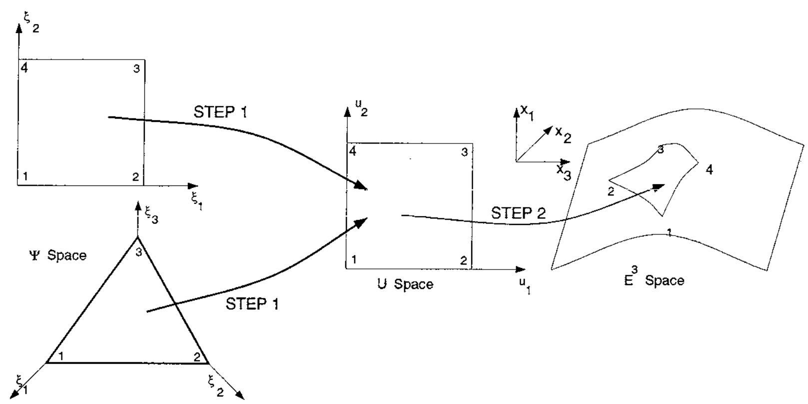

Figure 3 Two-step mapping from Ψ to $\mathbf{E^3}$ space

2.1 Mapping Finite Element space to Model Parametric space

A linear mapping is used to transform the coordinates in the Ψ space to the $\mathbf{U}$ space of the geometric model. The underlying assumption being that the finite element edge maps to a straight line in the parametric space of the model, $\mathbf{U}$. This assumption only works with untrimmed model faces because the trimming boundaries of the model face may not be linear in the **U** space and as a result of this, all finite element edges classified on the trimming edge will not map to a straight line. The linear mapping may be written as

$$\mathbf{U} = A + B * \Psi \tag{3}$$

The coefficients A and B will depend upon the type and the geometric classification of the finite element entity.

2.1.1 Mapping Finite Element Edges

The finite element edges will depend upon their classification with respect to the model. The model parametric space used to map the finite element edge will be the **U** space of the model entity on which the edge is classified.

2.1.1.1 Finite Element Edge Classified on Model Edge

For finite element edge classified on a model edge

$$\begin{aligned} \mathbf{u} &= \{u_1\} \\ \xi &= \{\xi_1\} \end{aligned} \tag{4}$$

and the mapping is given by

$$u_1 = (u_1^2 - u_1^1)\xi_1 \tag{5}$$

where u_1^1, u_1^2 are the coordinates of the finite element edge in the **U** space of the model edge on which it is classified.

2.1.1.2 Finite Element Edge Classified on Model Face

For finite element edge classified on a model face

$$\begin{aligned} \mathbf{u} &= \{u_1, u_2\} \\ \xi &= \{\xi_1\} \end{aligned} \tag{6}$$

and the mapping is given by

$$\begin{aligned} u_1 &= u_1^1 + (u_1^2 - u_1^1)\xi_1 \\ u_2 &= u_2^1 + (u_2^2 - u_2^1)\xi_1 \end{aligned} \tag{7}$$

2.1.2 Mapping Finite Element Faces

For a triangular finite element face with barycentric coordinates the mapping is given by

$$\mathbf{u} = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \underbrace{\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}}_{\mathbf{B}} \begin{Bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{Bmatrix} \tag{8}$$

The mapping coefficients b_{ij} can be obtained using the coordinates of the nodes in the **U** space of the geometric model.

$$B = \begin{bmatrix} u_1^1 & u_1^2 & u_1^3 \\ u_2^1 & u_2^2 & u_2^3 \end{bmatrix} \tag{9}$$

where u_j^i refers to the *j*th **U** space coordinate of the *i*th node in the element.

If the origin of the ψ space is fixed at node 1 then the mapping of a quadrilateral element is given by

$$\mathbf{u} = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} \sum_{i=1}^{4} N^i u_1^i \\ \sum_{i=1}^{4} N^i u_2^i \end{Bmatrix} \tag{10}$$

where

$$\begin{aligned} N^1 &= (1-\xi_1)(1-\xi_2) \\ N^2 &= \xi_1(1-\xi_2) \\ N^3 &= \xi_1\xi_2 \\ N^4 &= (1-\xi_1)\xi_2 \end{aligned} \tag{11}$$

3 Derivatives

The required derivatives are obtained using the chain rule.

$$\frac{\partial \mathbf{X}}{\partial \xi} = \left(\frac{\partial \mathbf{X}}{\partial \mathbf{u}}\right)\frac{\partial \mathbf{u}}{\partial \xi} \tag{12}$$

and

$$\begin{aligned}\frac{\partial^2 \mathbf{X}}{\partial \xi^2} &= \frac{\partial}{\partial \xi}\left\{\frac{\partial \mathbf{X}}{\partial \mathbf{u}}\frac{\partial \mathbf{u}}{\partial \xi}\right\} \\ &= \frac{\partial \mathbf{u}}{\partial \xi}\frac{\partial^2 \mathbf{X}}{\partial \mathbf{u}^2}\frac{\partial \mathbf{u}}{\partial \xi} + \frac{\partial \mathbf{X}}{\partial \mathbf{u}}\frac{\partial^2 \mathbf{u}}{\partial \xi^2}\end{aligned} \tag{13}$$

Note that $\frac{\partial \mathbf{X}}{\partial \mathbf{u}}, \frac{\partial^2 \mathbf{X}}{\partial \mathbf{u}^2}$ are the gradient and the Hessian respectively, of the model geometry with respect to the parametric space of the model. The gradient and the Hessian is typically available from the geometric modeling system. $\frac{\partial \mathbf{u}}{\partial \xi}$ represents the Jacobian of the constructed mapping between $\mathbf{U}$ and Ψ.

3.1 Derivatives for Finite Element Edge

The derivatives for individual finite elements are obtained based on the type of the finite element entity and its calssification with respect to the geometric model.

3.1.1 Finite Element Edge Classified on Model Edge

The Jacobian of the constructed mapping is a scalar term in this case and can be obtained by differentiating equation 5 as $\frac{\partial u_1}{\partial \xi_1} = u_1^2 - u_1^1$. The first and second derivatives can now be written as

$$\frac{\partial \mathbf{X}}{\partial \xi} = \left\{\begin{array}{c}\frac{\partial x_1}{\partial \xi_1} \\ \frac{\partial x_2}{\partial \xi_1} \\ \frac{\partial x_3}{\partial \xi_1}\end{array}\right\} = \left\{\begin{array}{c}\frac{\partial x_1}{\partial u_1} \\ \frac{\partial x_2}{\partial u_1} \\ \frac{\partial x_3}{\partial u_1}\end{array}\right\} * \frac{\partial u_1}{\partial \xi_1} \tag{14}$$

$$\frac{\partial^2 \mathbf{X}}{\partial \xi^2} = \left\{\begin{array}{c}\frac{\partial^2 x_1}{\partial \xi_1^2} \\ \frac{\partial^2 x_2}{\partial \xi_1^2} \\ \frac{\partial^2 x_3}{\partial \xi_1^2}\end{array}\right\} = \frac{\partial u_1}{\partial \xi_1} * \left\{\begin{array}{c}\frac{\partial^2 x_1}{\partial u_1^2} \\ \frac{\partial^2 x_2}{\partial u_1^2} \\ \frac{\partial^2 x_3}{\partial u_1^2}\end{array}\right\} * \frac{\partial u_1}{\partial \xi_1} \tag{15}$$

3.1.2 Finite Element Edge Classified on Model Face

The Jacobian of the constructed mapping is obtained by differentiating equation 7 as

$$\frac{\partial \mathbf{u}}{\partial \xi} = \begin{Bmatrix} \frac{\partial u_1}{\partial \xi_1} \\ \frac{\partial u_2}{\partial \xi_1} \end{Bmatrix} = \begin{Bmatrix} u_1^2 - u_1^1 \\ u_2^2 - u_2^1 \end{Bmatrix} \tag{16}$$

. The first and second derivatives can now be written as

$$\begin{aligned} \frac{\partial \mathbf{X}}{\partial \xi} = \begin{Bmatrix} \frac{\partial x_1}{\partial \xi_1} \\ \frac{\partial x_2}{\partial \xi_1} \\ \frac{\partial x_3}{\partial \xi_1} \end{Bmatrix} &= \begin{Bmatrix} \frac{\partial x_1}{\partial u_1}\frac{\partial u_1}{\partial \xi_1} + \frac{\partial x_1}{\partial u_2}\frac{\partial u_2}{\partial \xi_1} \\ \frac{\partial x_2}{\partial u_1}\frac{\partial u_1}{\partial \xi_1} + \frac{\partial x_2}{\partial u_2}\frac{\partial u_2}{\partial \xi_1} \\ \frac{\partial x_3}{\partial u_1}\frac{\partial u_1}{\partial \xi_1} + \frac{\partial x_3}{\partial u_2}\frac{\partial u_2}{\partial \xi_1} \end{Bmatrix} \\ &= \underbrace{\begin{bmatrix} \frac{\partial x_1}{\partial u_1} & \frac{\partial x_1}{\partial u_2} \\ \frac{\partial x_2}{\partial u_1} & \frac{\partial x_2}{\partial u_2} \\ \frac{\partial x_3}{\partial u_1} & \frac{\partial x_3}{\partial u_2} \end{bmatrix}}_{\mathbf{A}} \begin{Bmatrix} \frac{\partial u_1}{\partial \xi_1} \\ \frac{\partial u_2}{\partial \xi_1} \end{Bmatrix} \end{aligned} \tag{17}$$

$$\begin{aligned} \frac{\partial^2 \mathbf{X}}{\partial \xi^2} = \begin{Bmatrix} \frac{\partial^2 x_1}{\partial \xi_1^2} \\ \frac{\partial^2 x_2}{\partial \xi_1^2} \\ \frac{\partial^2 x_3}{\partial \xi_1^2} \end{Bmatrix} &= \begin{Bmatrix} \frac{\partial}{\partial \xi_1}\left(\frac{\partial x_1}{\partial u_1}\frac{\partial u_1}{\partial \xi_1} + \frac{\partial x_1}{\partial u_2}\frac{\partial u_2}{\partial \xi_1}\right) \\ \frac{\partial}{\partial \xi_1}\left(\frac{\partial x_2}{\partial u_1}\frac{\partial u_1}{\partial \xi_1} + \frac{\partial x_2}{\partial u_2}\frac{\partial u_2}{\partial \xi_1}\right) \\ \frac{\partial}{\partial \xi_1}\left(\frac{\partial x_3}{\partial u_1}\frac{\partial u_1}{\partial \xi_1} + \frac{\partial x_3}{\partial u_2}\frac{\partial u_2}{\partial \xi_1}\right) \end{Bmatrix} \\ &= \underbrace{\begin{bmatrix} \frac{\partial^2 x_1}{\partial u_1^2} & \frac{\partial^2 x_1}{\partial u_1 u_2} & \frac{\partial^2 x_1}{\partial u_2^2} \\ \frac{\partial^2 x_2}{\partial u_1^2} & \frac{\partial^2 x_2}{\partial u_1 u_2} & \frac{\partial^2 x_2}{\partial u_2^2} \\ \frac{\partial^2 x_3}{\partial u_1^2} & \frac{\partial^2 x_3}{\partial u_1 u_2} & \frac{\partial^2 x_3}{\partial u_2^2} \end{bmatrix}}_{\mathbf{H}} \begin{Bmatrix} \frac{\partial u_1}{\partial \xi_1}\frac{\partial u_1}{\partial \xi_1} \\ 2\frac{\partial u_1}{\partial \xi_1}\frac{\partial u_2}{\partial \xi_1} \\ \frac{\partial u_2}{\partial \xi_1}\frac{\partial u_2}{\partial \xi_1} \end{Bmatrix} \end{aligned} \tag{18}$$

The matrices **A** and **H** are the gradient and the Hessian respectively.

3.2 Derivatives of Finite Element Faces The derivatives for a triangular finite element face are given as

$$\frac{\partial \mathbf{X}}{\partial \xi} = \begin{bmatrix} \begin{Bmatrix} \frac{\partial \mathbf{X}}{\partial \xi_1} \end{Bmatrix} & \begin{Bmatrix} \frac{\partial \mathbf{X}}{\partial \xi_2} \end{Bmatrix} & \begin{Bmatrix} \frac{\partial \mathbf{X}}{\partial \xi_3} \end{Bmatrix} \end{bmatrix} \tag{19}$$

$$\frac{\partial^2 \mathbf{X}}{\partial \xi^2} = \begin{bmatrix} \begin{Bmatrix} \frac{\partial^2 \mathbf{X}}{\partial \xi_1^2} \end{Bmatrix} & \begin{Bmatrix} \frac{\partial^2 \mathbf{X}}{\partial \xi_1 \xi_2} \end{Bmatrix} & \begin{Bmatrix} \frac{\partial^2 \mathbf{X}}{\partial \xi_1 \xi_3} \end{Bmatrix} \\ \begin{Bmatrix} \frac{\partial^2 \mathbf{X}}{\partial \xi_2 \xi_1} \end{Bmatrix} & \begin{Bmatrix} \frac{\partial^2 \mathbf{X}}{\partial \xi_2^2} \end{Bmatrix} & \begin{Bmatrix} \frac{\partial^2 \mathbf{X}}{\partial \xi_2 \xi_3} \end{Bmatrix} \\ \begin{Bmatrix} \frac{\partial^2 \mathbf{X}}{\partial \xi_3 \xi_1} \end{Bmatrix} & \begin{Bmatrix} \frac{\partial^2 \mathbf{X}}{\partial \xi_3 \xi_2} \end{Bmatrix} & \begin{Bmatrix} \frac{\partial^2 \mathbf{X}}{\partial \xi_3^2} \end{Bmatrix} \end{bmatrix} \tag{20}$$

Note that each term in the above matrices represents a vector. The expressions for each individual vector has been worked out later.

The terms of the Jacobian of the constructed mapping for a triangular element are all constant obtained by differentiating equation 8

$$J = \begin{bmatrix} \frac{\partial u_1}{\partial \xi_1} & \frac{\partial u_1}{\partial \xi_2} & \frac{\partial u_1}{\partial \xi_3} \\ \frac{\partial u_2}{\partial \xi_1} & \frac{\partial u_2}{\partial \xi_2} & \frac{\partial u_2}{\partial \xi_3} \end{bmatrix} = \begin{bmatrix} u_1^1 & u_1^2 & u_1^3 \\ u_2^1 & u_2^2 & u_2^3 \end{bmatrix} \tag{21}$$

The derivatives for the quadrilateral element are given by

$$\frac{\partial \mathbf{X}}{\partial \xi} = \left[\left\{ \frac{\partial \mathbf{X}}{\partial \xi_1} \right\} \quad \left\{ \frac{\partial \mathbf{X}}{\partial \xi_2} \right\} \right] \tag{22}$$

$$\frac{\partial^2 \mathbf{X}}{\partial \xi^2} = \begin{bmatrix} \left\{ \frac{\partial^2 \mathbf{X}}{\partial \xi_1^2} \right\} & \left\{ \frac{\partial^2 \mathbf{X}}{\partial \xi_1 \xi_2} \right\} \\ \left\{ \frac{\partial^2 \mathbf{X}}{\partial \xi_2 \xi_1} \right\} & \left\{ \frac{\partial^2 \mathbf{X}}{\partial \xi_2^2} \right\} \end{bmatrix} \tag{23}$$

The terms of the Jacobian of the quadrilateral element mapping are not constants unlike the mapping for the triangular face elements. The Jacobian is obtained by differentiating equation 10 as

$$J = \begin{bmatrix} \frac{\partial u_1}{\partial \xi_1} & \frac{\partial u_1}{\partial \xi_2} \\ \frac{\partial u_2}{\partial \xi_1} & \frac{\partial u_2}{\partial \xi_2} \end{bmatrix} \tag{24}$$

where

$$\begin{aligned} \frac{\partial u_1}{\partial \xi_1} &= (u_1^2 - u_1^1) + \xi_2(u_1^1 - u_1^2 + u_1^3 - u_1^4) \\ \frac{\partial u_1}{\partial \xi_2} &= (u_1^4 - u_1^1) + \xi_1(u_1^1 - u_1^2 + u_1^3 - u_1^4) \\ \frac{\partial u_2}{\partial \xi_1} &= (u_2^2 - u_2^1) + \xi_2(u_2^1 - u_2^2 + u_2^3 - u_2^4) \\ \frac{\partial u_2}{\partial \xi_2} &= (u_1^4 - u_1^1) + \xi_1(u_1^1 - u_1^2 + u_1^3 - u_1^4) \end{aligned} \tag{25}$$

The individual vectors of the derivative matrices can now be worked out in detail using the gradient (**A**), the Hessian (**H**) and the appropriate terms from the Jacobian.

$$\frac{\partial \mathbf{X}}{\partial \xi_1} = [\mathbf{A}] \left\{ \begin{matrix} \frac{\partial u_1}{\partial \xi_1} \\ \frac{\partial u_2}{\partial \xi_1} \end{matrix} \right\} ; \ \frac{\partial \mathbf{X}}{\partial \xi_2} = [\mathbf{A}] \left\{ \begin{matrix} \frac{\partial u_1}{\partial \xi_2} \\ \frac{\partial u_2}{\partial \xi_2} \end{matrix} \right\} ; \ \frac{\partial \mathbf{X}}{\partial \xi_3} = [\mathbf{A}] \left\{ \begin{matrix} \frac{\partial u_1}{\partial \xi_3} \\ \frac{\partial u_2}{\partial \xi_3} \end{matrix} \right\} \tag{26}$$

$$\frac{\partial^2 \mathbf{X}}{\partial \xi_1^2} = [\mathbf{H}] \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} ; \frac{\partial^2 \mathbf{X}}{\partial \xi_1 \xi_2} = [\mathbf{H}] \begin{Bmatrix} d \\ e \\ f \end{Bmatrix} ; \frac{\partial^2 \mathbf{X}}{\partial \xi_1 \xi_3} = [\mathbf{H}] \begin{Bmatrix} g \\ h \\ i \end{Bmatrix}$$

$$\frac{\partial^2 \mathbf{X}}{\partial \xi_2 \xi_1} = \frac{\partial^2 \mathbf{X}}{\partial \xi_1 \xi_2} ; \frac{\partial^2 \mathbf{X}}{\partial \xi_2^2} = [\mathbf{H}] \begin{Bmatrix} j \\ k \\ l \end{Bmatrix} ; \frac{\partial^2 \mathbf{X}}{\partial \xi_2 \xi_3} = [\mathbf{H}] \begin{Bmatrix} m \\ n \\ o \end{Bmatrix} \tag{27}$$

$$\frac{\partial^2 \mathbf{X}}{\partial \xi_3 \xi_1} = \frac{\partial^2 \mathbf{X}}{\partial \xi_1 \xi_3} ; \frac{\partial^2 \mathbf{X}}{\partial \xi_3 \xi_2} = \frac{\partial^2 \mathbf{X}}{\partial \xi_2 \xi_3} ; \frac{\partial^2 \mathbf{X}}{\partial \xi_3^2} = [\mathbf{H}] \begin{Bmatrix} p \\ q \\ r \end{Bmatrix}$$

where

$$\begin{aligned}
a &= \frac{\partial u_1}{\partial \xi_1}\frac{\partial u_1}{\partial \xi_1} ; \; b = \frac{\partial u_1}{\partial \xi_1}\frac{\partial u_2}{\partial \xi_1} + \frac{\partial u_1}{\partial \xi_1}\frac{\partial u_2}{\partial \xi_1} ; \; c = \frac{\partial u_2}{\partial \xi_1}\frac{\partial u_2}{\partial \xi_1} \\
d &= \frac{\partial u_1}{\partial \xi_1}\frac{\partial u_1}{\partial \xi_2} ; \; e = \frac{\partial u_1}{\partial \xi_1}\frac{\partial u_2}{\partial \xi_2} + \frac{\partial u_2}{\partial \xi_1}\frac{\partial u_1}{\partial \xi_2} ; \; f = \frac{\partial u_2}{\partial \xi_1}\frac{\partial u_2}{\partial \xi_2} \\
g &= \frac{\partial u_1}{\partial \xi_1}\frac{\partial u_1}{\partial \xi_3} ; \; h = \frac{\partial u_1}{\partial \xi_1}\frac{\partial u_2}{\partial \xi_3} + \frac{\partial u_2}{\partial \xi_1}\frac{\partial u_1}{\partial \xi_3} ; \; i = \frac{\partial u_2}{\partial \xi_1}\frac{\partial u_2}{\partial \xi_3} \\
j &= \frac{\partial u_1}{\partial \xi_2}\frac{\partial u_1}{\partial \xi_2} ; \; k = \frac{\partial u_1}{\partial \xi_2}\frac{\partial u_2}{\partial \xi_2} + \frac{\partial u_1}{\partial \xi_2}\frac{\partial u_2}{\partial \xi_2} ; \; l = \frac{\partial u_2}{\partial \xi_2}\frac{\partial u_2}{\partial \xi_2} \\
m &= \frac{\partial u_1}{\partial \xi_2}\frac{\partial u_1}{\partial \xi_3} ; \; n = \frac{\partial u_1}{\partial \xi_2}\frac{\partial u_2}{\partial \xi_3} + \frac{\partial u_2}{\partial \xi_2}\frac{\partial u_1}{\partial \xi_3} ; \; o = \frac{\partial u_2}{\partial \xi_2}\frac{\partial u_2}{\partial \xi_3} \\
p &= \frac{\partial u_1}{\partial \xi_3}\frac{\partial u_1}{\partial \xi_3} ; \; q = \frac{\partial u_1}{\partial \xi_3}\frac{\partial u_2}{\partial \xi_3} + \frac{\partial u_1}{\partial \xi_3}\frac{\partial u_2}{\partial \xi_3} ; \; r = \frac{\partial u_2}{\partial \xi_3}\frac{\partial u_2}{\partial \xi_3}
\end{aligned} \tag{28}$$

4 Implementation and Results

The mapping function has been designed as stand alone operators which can be called by application routines which need the position vectors and/or the derivatives for individual finite elements. The description and the arguments of the operators are given below

1. EVALPT : Operator to return the position vector in $\mathbf{E^3}$ space for given coordinates in the Ψ space of the finite element.
Fortran syntax:

```
FT_EVALPT(TYPE,FENT,PARM,RXYZ)
INTEGER TYPE,FENT
DOUBLE PRECISION PARM(3),RXYZ(3)
```

C syntax:

```
void ft_evalpt(int *type,int *fent,
               double *parm,double *rxyz)
```

where

TYPE : Type of finite element (edge/face) [input]
FENT : Finite Element entity [input]
PARM : Coordinates in Ψ space [input]
RXYZ : Coordinates in $\mathbf{E^3}$ [output]

2. EVALDV : Operator to return the derivative vectors in $\mathbf{E^3}$ space at given coordinates in the Ψ space of the finite element upto the second order.
Fortran syntax:

```
FT_EVALDV(TYPE,FENT,ORDER,PARM,DERV)
INTEGER TYPE,FENT,ORDER
DOUBLE PRECISION PARM(3),DERV(*)
```

C syntax:

```
void ft_evalpt(int *type,int *fent,
               int *order,double *parm,double *derv)
```

where

TYPE : Type of finite element (edge/face) [input]

FENT : Finite Element entity [input]
PARM : Coordinates in Ψ space [input]
DERV : Derivatives in $\mathbf{E}^3$ space [output]. The derivatives are returned in a 1-dimensional array. For finite element edges, the first and the second derivatives are returned in the first three locations of the array

$$\begin{aligned} \frac{\partial x_1}{\partial \xi_1}, \frac{\partial^2 x_1}{\partial \xi_1^2} &= derv[0] \\ \frac{\partial x_2}{\partial \xi_1}, \frac{\partial^2 x_2}{\partial \xi_1^2} &= derv[1] \\ \frac{\partial x_3}{\partial \xi_1}, \frac{\partial^2 x_3}{\partial \xi_1^2} &= derv[2] \end{aligned} \tag{29}$$

For the triangular finite element faces, array location 0-8 contain the components of the three gradient vectors (order=1); for second order derivatives, indices 0-2 contain components of $\frac{\partial^2 \mathbf{X}}{\partial \xi_1^2}$, indices 3-5 contain components of $\frac{\partial^2 \mathbf{X}}{\partial \xi_1 \xi_2}$, indices 6-8 contain components of $\frac{\partial^2 \mathbf{X}}{\partial \xi_1 \xi_3}$, indices 9-11 contain components of $\frac{\partial^2 \mathbf{X}}{\partial \xi_2 \xi_1}$, indices 12-14 contain components of $\frac{\partial^2 \mathbf{X}}{\partial \xi_2^2}$, indices 15-17 contain components of $\frac{\partial^2 \mathbf{X}}{\partial \xi_2 \xi_3}$, indices 18-20 contain components of $\frac{\partial^2 \mathbf{X}}{\partial \xi_3 \xi_1}$, indices 21-23 contain components of $\frac{\partial^2 \mathbf{X}}{\partial \xi_3 \xi_2}$, indices 24-26 contain components of $\frac{\partial^2 \mathbf{X}}{\partial \xi_3^2}$. For quadrilateral finite element faces, array location 0-5 contain the components of the three gradient vectors (order=1); for second order derivatives, indices 0-2 contain components of $\frac{\partial^2 \mathbf{X}}{\partial \xi_1^2}$, indices 3-5 contain components of $\frac{\partial^2 \mathbf{X}}{\partial \xi_1 \xi_2}$, indices 6-8 contain components of $\frac{\partial^2 \mathbf{X}}{\partial \xi_2 \xi_1}$, indices 9-11 contain components of $\frac{\partial^2 \mathbf{X}}{\partial \xi_2^2}$.

The operators have been implemented to work with the Finite Octree mesh generator using Parasolid and Scorec3D modelers.

Figure 4 shows the mapping for shell meshes generated within Finite Octree using the Scorec3D and the Parasolid modeler. Note that the mapping of the faces is depicted by drawing the family of curves representing $\xi_1 = constant$.

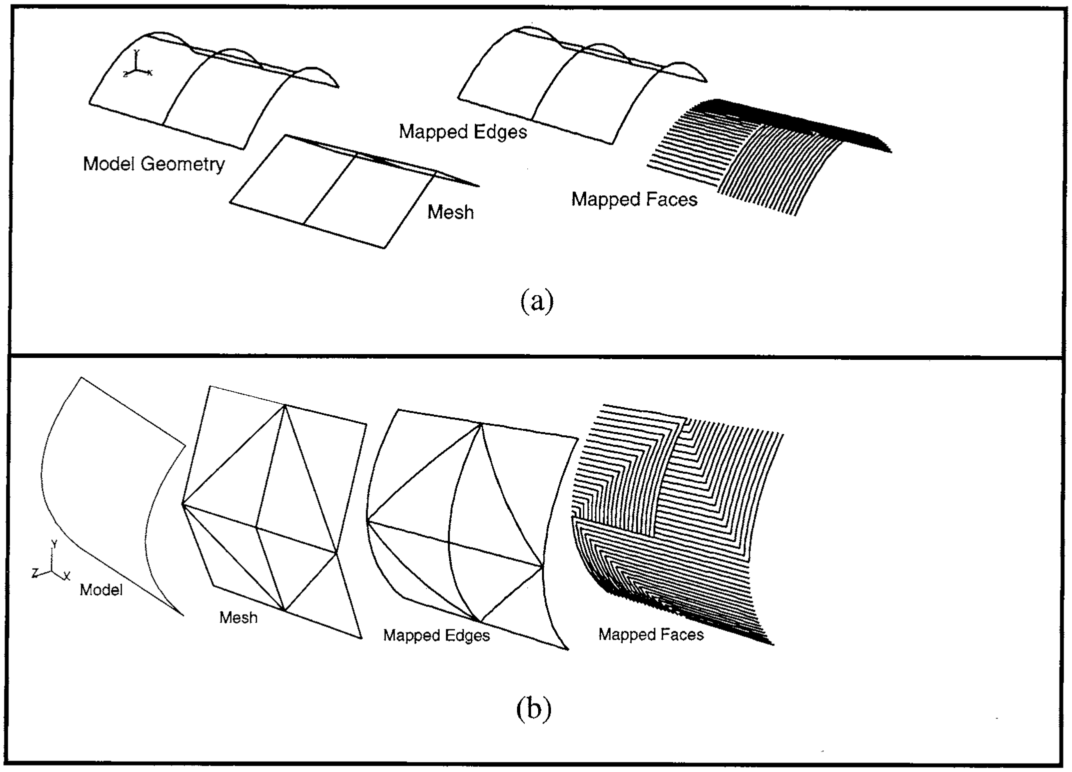

Figure 4 (a) Quadrilateral mesh (Scorec3D) (b) Triangular mesh (Parasolid)

Bibliography

[1] B. A. Szabo and I. Babuska. *Finite Element Analysis*. Wiley Interscience, New York, 1991.