AUTOMATIC MESH GENERATION OF COMPLEX CONFIGURATIONS INCLUDING VISCOUS BOUNDARY LAYERS

R. Garimella[1], M.S. Shephard[2], B.E. Webster[3]

ABSTRACT

The problem of generating unstructured tetrahedral meshes for viscous flow simulations in complex non-manifold domains starting from a geometric model is addressed. The method adopted to generate the boundary layer mesh is the Generalized Advancing Layers technique incorporating a number of features essential for the creation of valid and good quality meshes. The procedures have been tested on a number of complex models such as vehicle interiors as well as vehicle exteriors with under-the-hood and under-the-body detail. The capabilities of the method will be illustrated by example.

INTRODUCTION

Viscous flow simulations require good resolution of the solution near model surfaces in boundary layers where gradients normal to the surface are very high relative to those in the tangential direction. Isotropic mesh refinements in such regions leads to an unacceptably large number of elements. The mesh must be highly anisotropic in the boundary layer region and transition to an isotropic mesh in the far field [10].

A mesh generator for such applications must:

1. generate layers of highly stretched elements on user-specified set of boundaries,
2. deal with complex geometric models including non-manifold domains [11],
3. transition smoothly between the anisotropic and isotropic parts of the mesh, and
4. generate valid and good quality meshes at sharp or narrow corners in the domain.

The mesh generation procedure used here starts with a surface mesh that is properly classified on a geometric model and is not self-intersecting. The boundary layer mesh is generated in the domain by the *Generalized Advancing Layers* method and consists of layers of anisotropic elements grown from the appropriate boundaries of the model. The rest of the domain is filled by an isotropic mesh generator capable of triangulating an arbitrary polyhedron with a boundary that is not self-intersecting. Finally the mesh is post-processed to improve its quality. The mesh optimization utilizes node reposition and local mesh modification procedures to improve the quality of the mesh [5].

Generalized Advancing Layers creates the mesh by generating boundary layer vertices along curves (called *growth curves*) originating from vertices of the surface mesh and connecting them to form layers of highly stretched tetrahedra along the model boundaries. The layers are the tetrahedronization of

1. triangular prisms grown on surface mesh faces [1][8][7][6], and
2. blend polyhedra on surface mesh edges, vertices at convex corners or areas of high curvature.

To ensure that the boundary layer tetrahedra are valid, the boundary layer points are repositioned as necessary before element creation. The repositioning process is a combination of smoothing of the growth curves along which the boundary layer vertices are placed and local shrinking of layers. Also, in cases where the boundary layer mesh is grown only on some faces, boundary layer entities may be incorporated into the adjacent surface triangulation before the solid element creation. Finally, any interference between different boundary layers is eliminated by local shrinking of layer thicknesses and local deletion of elements.

SURFACE MESH GENERATION

The surface mesh generation procedure [2][3] discretizes general 3D surfaces using a 2D constrained Delaunay triangulation algorithm that is modified to account for the mapping from parametric space to the real space. The mesh generator is capable of meshing general geometries quickly with good control over mesh gradation and automatic refinement methods such as curvature based refinement methods.

BOUNDARY LAYER MESH GENERATION

Starting from a surface mesh of the geometric model, each mesh vertex classified on a boundary model entity (model vertex, face or edge) require to have a boundary layer, points are generated along one or more

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[1] Graduate Research Assistant, Phone: (518)-276-6795. E-Mail: garimell@scorec.rpi.edu

[2] Director, Scientific Computation Research Center, 7017 CII, Rensselaer Polytechnic Institute, Troy, NY, 12180-3590 Phone: (518) 276-6795. E-Mail: shephard@scorec.rpi.edu

[3] Simmetrix Corp., Phone: (313) 248-3161. E-Mail: bwebster@simmetrix.com

growth curves. Node locations for the boundary layer mesh are determined according to user specifications. Layer thicknesses are prescribed to increase geometrically as t, tr, tr^2, tr^3, $tr^{(n-1)}$ or exponentially as t, t^2, t^3, $t^{(n-1)}$, where t is the first layer thickness, r is a stretching factor and n is the number of layers. These points form the vertices of triangular prisms and blend polygons that are used to form regions (Figure 1a,b).

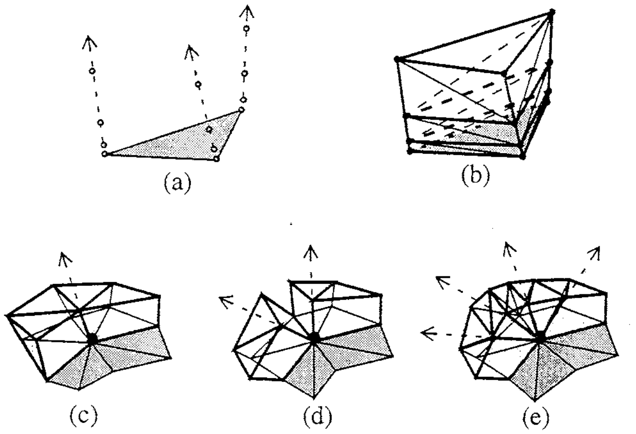

Figure 1 (a) Growth curves from face vertices, (b) Triangular prism on mesh face, (c) Single growth curve at vertex and resulting prisms, (d) multiple growth curves at vertex with prisms, (e) multiple growth curves with prisms.

The number and shape of growth curves at a mesh vertex is influenced by the classification of the vertex, dihedral angles between connected mesh faces, model face normals, curvature and topology with respect to 2-manifold or non-manifold.

Multiple growth curves may be required at mesh vertices to generate valid meshes at non-manifold interfaces. Material interfaces and embedded model faces are examples of situations which require multiple growth curves. The determination of the number of growth curves necessary for generating a valid boundary layer mesh is facilitated by the radial edge representation of the non-manifold geometric model [11].

Multiple growth curves are also required at mesh vertices on convex corners or in areas of high curvature. At such a mesh vertex, use of a common set of nodes for all the prisms on faces connected to the mesh vertex leads to excessive distortions of the prisms. The use of multiple sets of nodes, one for each set of faces with small deviation between their normals allows creation of good quality prisms (Figure 1d).

If all the prisms on mesh faces connected to a vertex do not share a common growth curve, gaps form between the prisms, exposing highly stretched faces to the isotropic mesh generator. To fill this gap, the Generalized Advancing Layers method creates boundary layer blend polyhedra which are then tetrahedronized. To maintain a smooth gradation in mesh sizes on the outer envelope of the boundary layer mesh, the blend polyhedra may utilize additional growth curves generated at the mesh vertex (Figure 1e).

Growth curves are straight lines in the interior of the model but may be of a general shape on the boundary. Growth curves are forced to lie on the boundary when the node locations on the growth curve are too close to a model face or edge where their proximity precludes the creation of well shaped elements later by the isotropic mesh generator. This can occur only when the growth curve originates from a mesh vertex classified on a model vertex or an edge and a boundary layer mesh is not requested on the entity the growth curve will lie on. Since the model surface may be of arbitrary shape, growth curves may lie only partly on boundary entity, joining a boundary entity in the lower layers and separating in the upper ones as the deviation of nodes from the model surface becomes too large. The locations of the boundary layer nodes on the model boundary is computed by a closest point search from its location on the straight line growth curve.

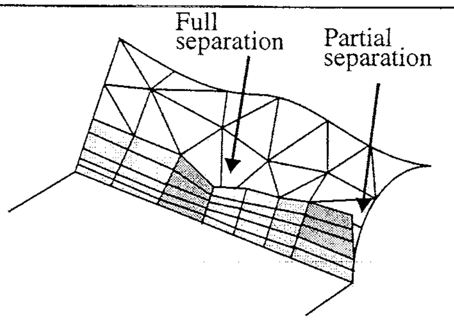

Figure 2 Partial and complete separation of boundary layer growth curves from adjacent model.

The creation of growth curves on model edges and faces must take into consideration the validity of the surface mesh after the appropriate boundary layer entities have been incorporated into it. Once the boundary growth curves have been fixed, the resulting vertices, edges and faces classified on the model boundary are created and the surface mesh modified to account for their presence. The surface mesh is retriangulated by an advancing front method.

FIXING CROSSOVER OF GROWTH CURVES

The initial generation of growth curves is done without consideration of whether growth curves in the neighborhood crossover or not. Growth directions can crossover at acute corners, or in the presence of strong curvature in the surfaces. Crossover of growth directions leads to inside-out prisms. Typical crossover situations are illustrated in 2D in Figure 3.

Crossover of growth curves is rectified by a combination of methods as outlined below:

Fixing crossover by smoothing is an iterative procedure in which each growth curve is recomputed to be the average of all its adjacent growth curves[1,2]. Growth curves on curved model faces are approximated by a vector from the base node to the top node. During smoothing, originally valid prisms are not allowed to become invalid.

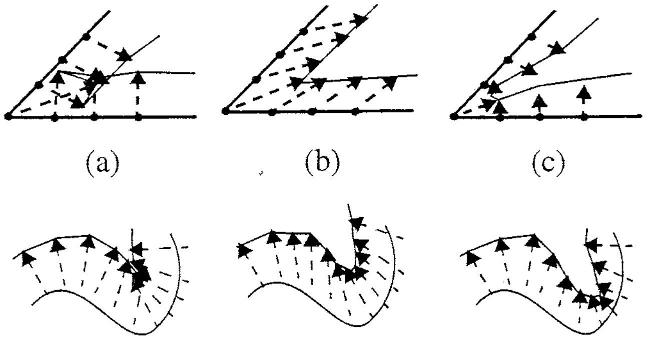

Figure 3 2D example of growth curve crossover (a), and its fix using smoothing (b), and compression (c).

If crossover of growth curves persists in some portions of the mesh, the layers are compressed locally. For each prism that is invalid, some or all of its growth curves are compressed so that the prism becomes valid while preserving the validity of adjacent prisms. If the prism can be made valid, the local reduction in the height of the boundary layer is propagated out recursively to the adjacent growth directions so as to grade the boundary layer heights smoothly. Multiple passes of the compression process are carried out to improve the chances of fixing the invalid prisms.

If any invalid prisms still remain after the smoothing and compression of growth curves, the appropriate growth curves are trimmed by reducing the number of nodes locally. This is equivalent to deleting invalid elements from the top of the boundary layer mesh. This is necessary in only very constrained situations.

Fixing crossover of adjacent growth curves on the model boundary requires additional care since growth curves may not be straight lines and the boundary layer quadrilaterals are not necessarily planar.

TETRAHEDRONIZATION

Once the surface mesh has been made conforming with the boundary layer mesh, interior boundary layer entities are formed. This process consists of (i) creating vertices and edges along interior growth curves, (ii) creating interior edges and faces, and (iii) creating tetrahedral elements.

The two templates for tetrahedronization of the triangular prism are shown in Figure 4. Since certain face diagonals can lead to configurations which cannot be tetrahedronized, care is taken to always generate valid diagonals. This ensured by assigning each surface mesh vertex a unique ID number and always constructing a face diagonal with its lower end on the growth curve from the vertex with the smaller ID.

Creation of blend meshes follows the same basic steps as the prism tetrahedronization. The blend meshes at model edges can still be generated by template like procedures where it is necessary to use other templates besides prisms and geometric criteria must be used for some connections in a variable edge blend mesh. Trivalent vertices satisfying criteria on the dihedral angles between the faces allow for a template to be used. On the other hand, filling voids at model vertices is more complex when more model faces use the model vertex. The use of the specialized element removal procedure is required at these model vertices.

INTERFERENCE DETECTION AND CORRECTION

When boundary layer meshes are generated in confined spaces in models, it is possible for layers from two different model faces, or different portions of the same model face, to interpenetrate each other. If not corrected, the input to the isotropic volume mesher will be a domain with a self-intersecting boundary.

The approach used here to detect and correct interpenetration of layers is a based on concepts from the advancing front method. Once the tetrahedral elements in the boundary layer mesh are generated, it is possible to identify a set of mesh faces which will form the front or the boundary of one or more polyhedral cavities to be filled in by an isotropic mesh generator (referred to as either *front faces* or *exposed faces*). Each exposed face is either:

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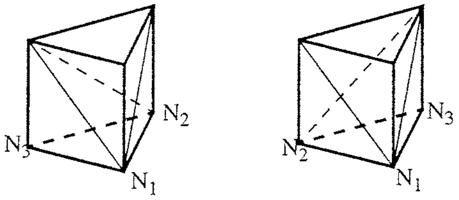
[1] All growth curves sharing a prism with a given growth curve are *adjacent* to the growth curve.

[2] Only growth curves in the interior are smoothed this way - a different procedure based on approximation of the growth curves by straight lines is adopted for boundary curves.



N_i = Vertex ID numbers, i = 1, 2, 3
$N_1 < N_2 < N_3$

Figure 4 Two templates used for prism tetrahedronization.

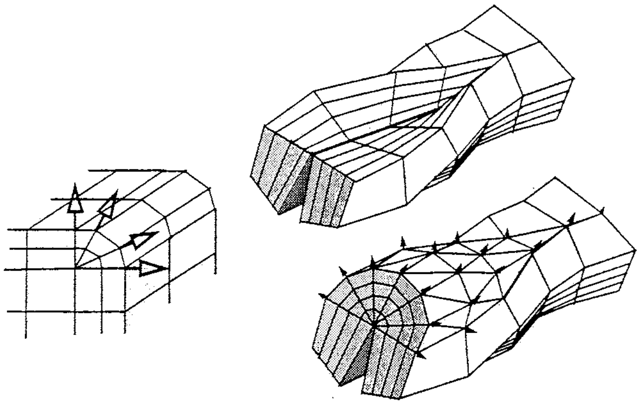


Figure 5 (a) Fixed boundary layer blend mesh (b) variable boundary layer blend mesh to fill the gap between prisms at model edge.

1. classified interior but with a mesh region only on one side,
2. classified on a 2-manifold model face but no mesh region connected to it, or
3. classified on a non-manifold model face used by two model regions but with one or no mesh regions connected to it.

The procedure checks for interference of exposed faces with each other. When an intersection is found it is fixed if possible by locally compressing the two boundary layers while keeping the adjacent prisms valid. This can cause some other exposed faces in the neighborhood to interfere with each other which must now be corrected. The procedure is therefore applied iteratively to the mesh until no exposed faces interfere with each other. This process is illustrated in 2D in Figure 6. The method is made efficient by checking exposed faces adjacent to those that have been moved thereby fixing interference in an entire neighborhood before moving on. In a typical problem, interference between layers is fixed in one or two iterations. If all attempts to fix the mesh by compressing the layers fail, then the interference is corrected by local deletion of elements. An octree [9] is used to make the search for front faces in a given neighborhood more efficient.

VOLUME FILLING BY ISOTROPIC MESH GENERATION

The volume mesher [3][4] can start from any set of polyhedral domains bounded by triangular mesh faces classified on the boundary or the interior of the model (the collection of these faces is called the *front*). The first step in the procedure is to build a variable level octree which reflects the mesh control information and is consistent with the triangulation on the boundary of the model and classify the octants as being interior, exterior or on the boundary. Interior octants are meshed using templates. Face removal procedures are then used to connect the boundary triangulation to the interior octants. Face removals are performed using the tree as a localization tool and also use the Delaunay criterion to speed-up the process of target selection.

MESH OPTIMIZATION

The mesh optimization procedure consists of large dihedral angle optimization followed by node-repositioning. The large dihedral angle procedure optimizes the mesh by applying local mesh modification procedures to an initial mesh in order to reach a target global quality measure. The various tools used are edge

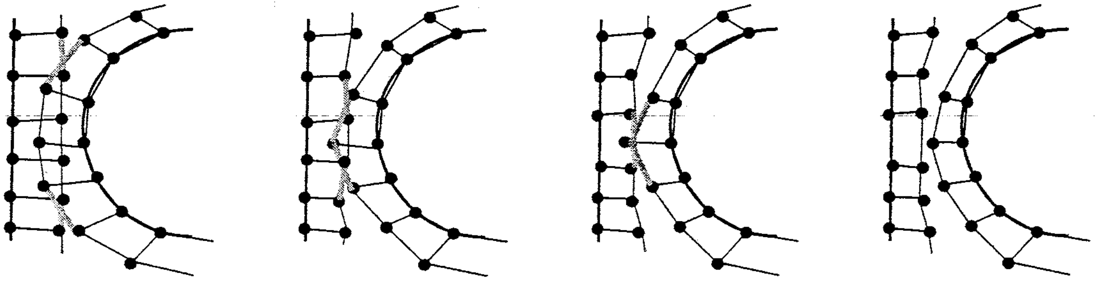


Figure 6 Fixing interference by compressing layers wherever exposed faces (shaded lines) intersect.

collapse, edge split, edge swap, multi-edge swap and split and collapse combinations. The node repositioning technique improves the distribution of points and the quality of the mesh by a constrained weighted Laplacian smoothing procedure. The boundary layer mesh remains constrained during the first pass of mesh optimization while the top layers are allowed to be modified in the next.

RESULTS

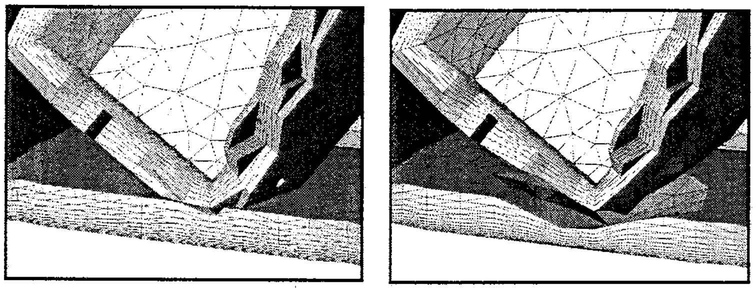


Figure 7 Resolving intersection of boundary layers by compression (a) Interpenetrating boundary layers (b) boundary layers compressed locally to fix problem

Figure 7 illustrates the capability of the Generalized Advancing Layers method to resolve intersections between boundary layers in a simple example.The boundary layers are shown interpenetrating in Figure 7(a) and the problem is shown fixed in Figure 7(b) by compression of layers. The smooth gradation of the neighboring prism heights up to the requested boundary layer thickness can also be seen in the picture.

Figure 8 shows the boundary layer mesh for the under-body of an automobile. The boundary layer mesh (in white) in this example was chosen to be thicker than normal for clarity of illustration. The mesh shown is valid, non-self intersecting and suitable for input to the isotropic mesh generator for completion of the mesh generation task. Using smoothing and compression of growth curves, the method has successfully resolved crossover of growth curves in areas of high curvature and interpenetration of the boundary layers in very confined spaces except in very few localized areas where elements had to be deleted.

CLOSING REMARKS

The Generalized Advancing Layers method for generating highly anisotropic meshes in the boundary layer regions of viscous flow domains was described. The method is designed to generate valid and good quality meshes of complex non-manifold domains starting from a geometric model. Essential geometric and topological considerations for creation of valid meshes by this technique are described. The procedures for correction of interference between boundary layers is described and demonstrated. The approach of growing boundary layers from the surface mesh by advancing layers is generalized with the introduction of blend boundary layers, necessary for creating good boundary layers over general domains. Results were presented demonstrating capabilities of the methodology to handle general configurations. Work is in progress to deal with the remaining types of non-manifold faces and with blend meshes for sharp corners.

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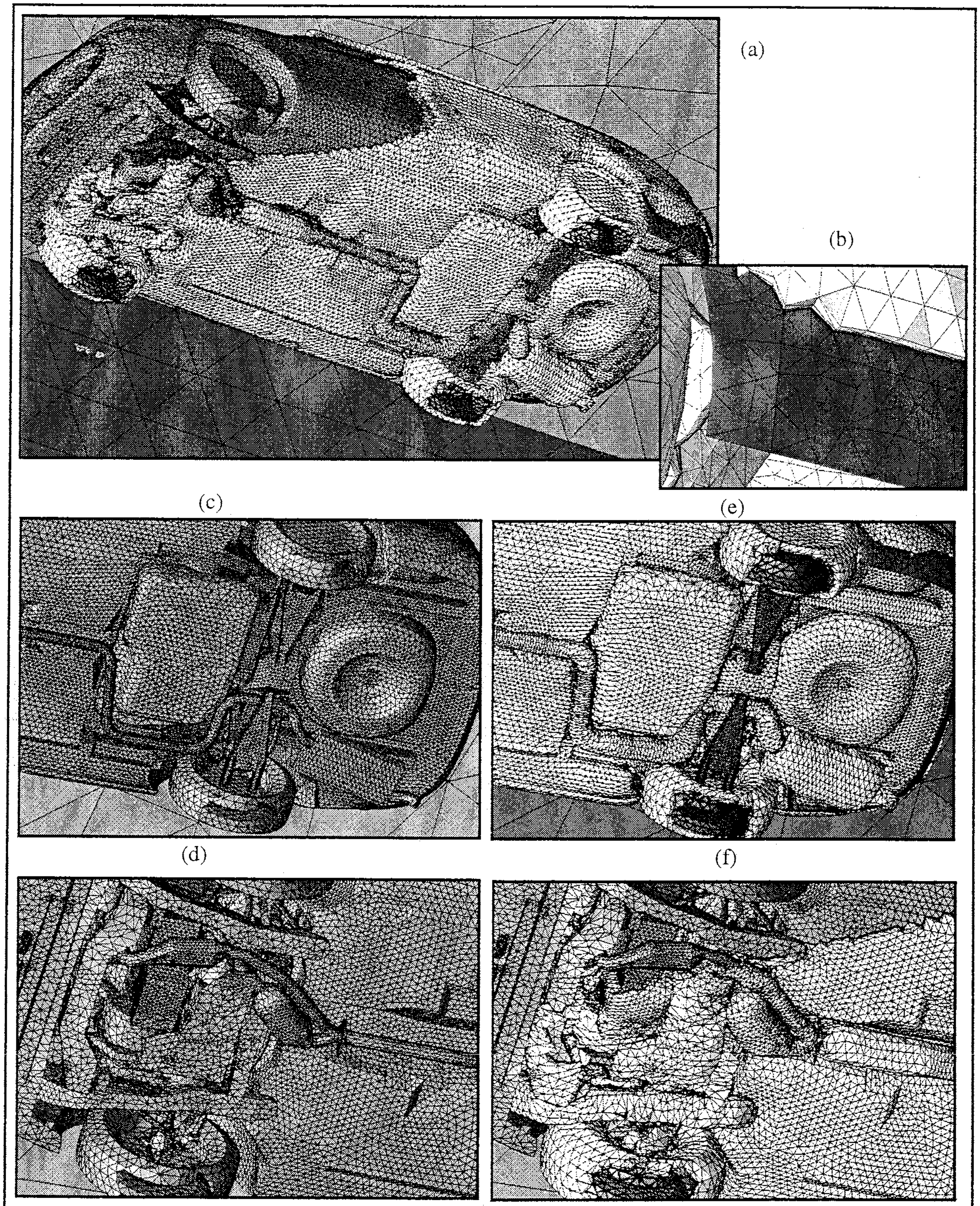

Figure 8 Boundary layer mesh on the under-body of an automobile. (a) Cut-away of boundary layer (b) Closeup of the layers (c),(d) Closeups of surface mesh (e),(f) boundary layer mesh on closeup areas

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