



## DISCRETE ELEMENT SIMULATIONS OF WATER FLOW THROUGH GRANULAR SOILS

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### ABSTRACT

This paper presents a coupled micro-mechanical technique to model pore water flow and solid phase deformation of granular soils. The fluid motion is idealized using averaged Navier-Stokes equations, and the discrete element method (DEM) is employed to model the assemblage of granular particles. The fluid-particle interactions are provided by established semi-empirical relationships. The developed model is used to simulate a range of seepage conditions through idealized granular samples. The conducted simulations are validated using published experimental results.

**Keywords:** Discrete elements, pore water flow, fluid dynamics, granular soils.

### INTRODUCTION

The dynamic flow of water and other viscous fluids through granular soils is commonly modeled using Darcy's law with time-independent permeability coefficients. Experimental studies show that this law is not valid for large nonlaminar fluid velocities which may develop under high hydraulic gradients (e.g., Burmister 1954). Furthermore, particle rearrangements leading to substantial variations in porosity are prevalent during soil liquefaction and other extreme flow driven phenomena, such as piping. These variations in porosity affect significantly the soil hydraulic conductivity and stiffness characteristics. Experimental investigations furnish only macroscopic phenomenological characterization of these complex response mechanism.

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Microscopic numerical analyses provide effective alternative tools to assess the hydraulic and deformation characteristics of granular soils in a coupled fashion.

Pore-scale models of fluid flow through porous media (e.g., Zhu et al. 1999) generally provide useful microscopic details such as drag forces and actual flow pattern, but present a significant computational challenge. Herein, pore water flow is idealized using averaged Navier-Stokes equations which are coupled with a discrete element model of the solid phase, or soil particulate matrix (Tsuji et al. 1993). Such coupled formulation is computationally tractable and provides adequate level of information on pore water flow. This paper presents preliminary results on an ongoing research effort to simulate the micro-mechanical fluid-particle response under deformation and extreme flow conditions such as liquefaction and piping.

## MICRO-MECHANICAL MODEL OF SATURATED GRANULAR SOILS

### Fluid Phase

The pore fluid motion is modeled using averaged Navier-Stokes equations (Anderson and Jackson 1967). For an inviscid incompressible fluid, this model consists of:

Continuity Equation:

$$\frac{\partial n}{\partial t} + \frac{\partial(nu_i)}{\partial x_i} = 0 \quad (1)$$

Momentum Equations:

$$\frac{\partial(nu_i)}{\partial t} + \frac{\partial(nu_i u_j)}{\partial x_j} = -\frac{n}{\rho} \frac{\partial p}{\partial x_i} + n\rho g_i + d_i \quad (2)$$

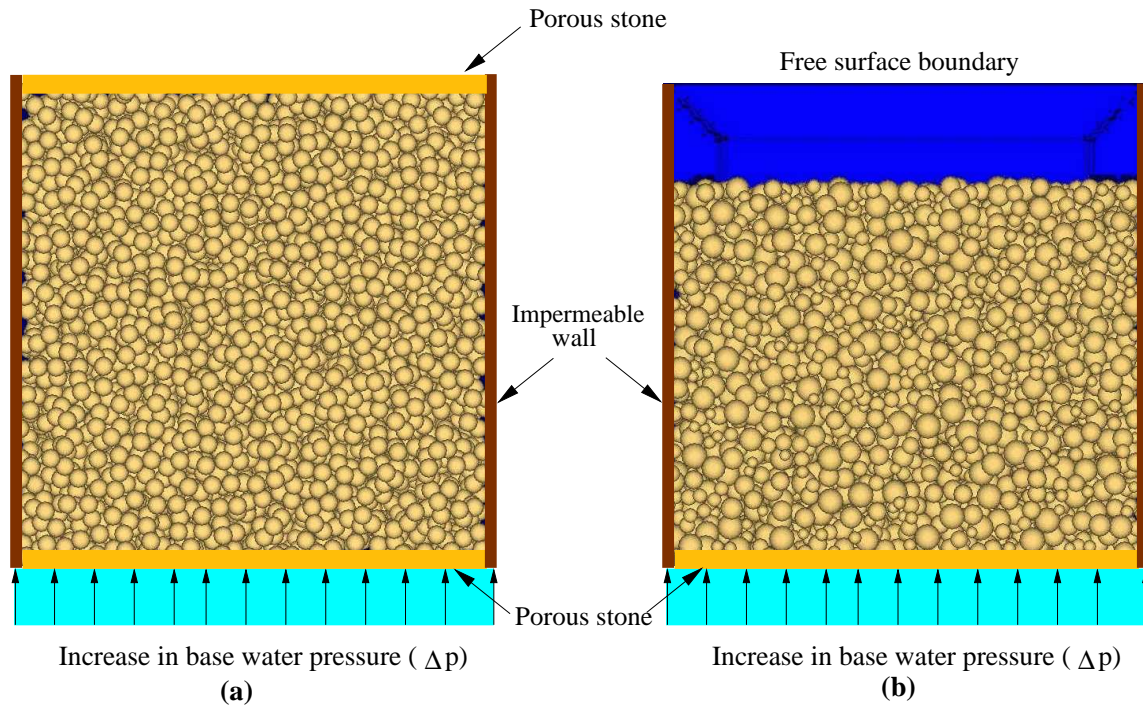
along with appropriate boundary and initial conditions. In Eqs. 1 and 2,  $x_i$  ( $i = 1, 2, 3$ ) are Cartesian coordinates,  $n$  is porosity,  $u_i$  is fluid velocity,  $\rho$  is fluid density,  $p$  is fluid pressure, and  $g_i$  is gravitational force per unit mass. The terms  $d_i$  ( $i = 1, 2, 3$ ) represent the resultant of the drag forces exerted by the fluid on the particles. Semi-empirical relationships are commonly used to quantify these interaction terms. The equations proposed by Ergun (1952), and Wen and Yu (1966) are used in this study. The averaged Navier-Stokes equations (Eqs. 1 and 2) are discretized using a finite volume technique on a staggered grid to ensure stability (Patankar 1980).

### Solid Phase

Granular soils consist of an assemblage of discontinuous particles which can be modeled effectively using the discrete element method, DEM (Cundall and Strack 1979). In DEM models the particles are rigid but can overlap. The interaction forces between any two particles are dictated by contact laws and are direct functions of the overlap and the relative movement at the contact. Details of the employed PFC3D DEM code and associated contact law model are given in Itasca (1999).

### Coupled Response

An explicit time-integration scheme is used to evaluate the coupled fluid-particle response. The flow domain is discretized into parallelepiped cells and averaged Navier-Stokes equations are solved using a finite volume technique (as mentioned above). Average drag forces are evaluated for each individual cell based on mean values of porosity, as well as of particle velocities and sizes within this cell. These drag forces are then applied to individual particles proportionally to their volumes. Deformation of the solid phase subjected to the drag forces along with any external loads is subsequently computed using the DEM technique.



**FIG. 1. Idealized soil samples**

## NUMERICAL SIMULATIONS

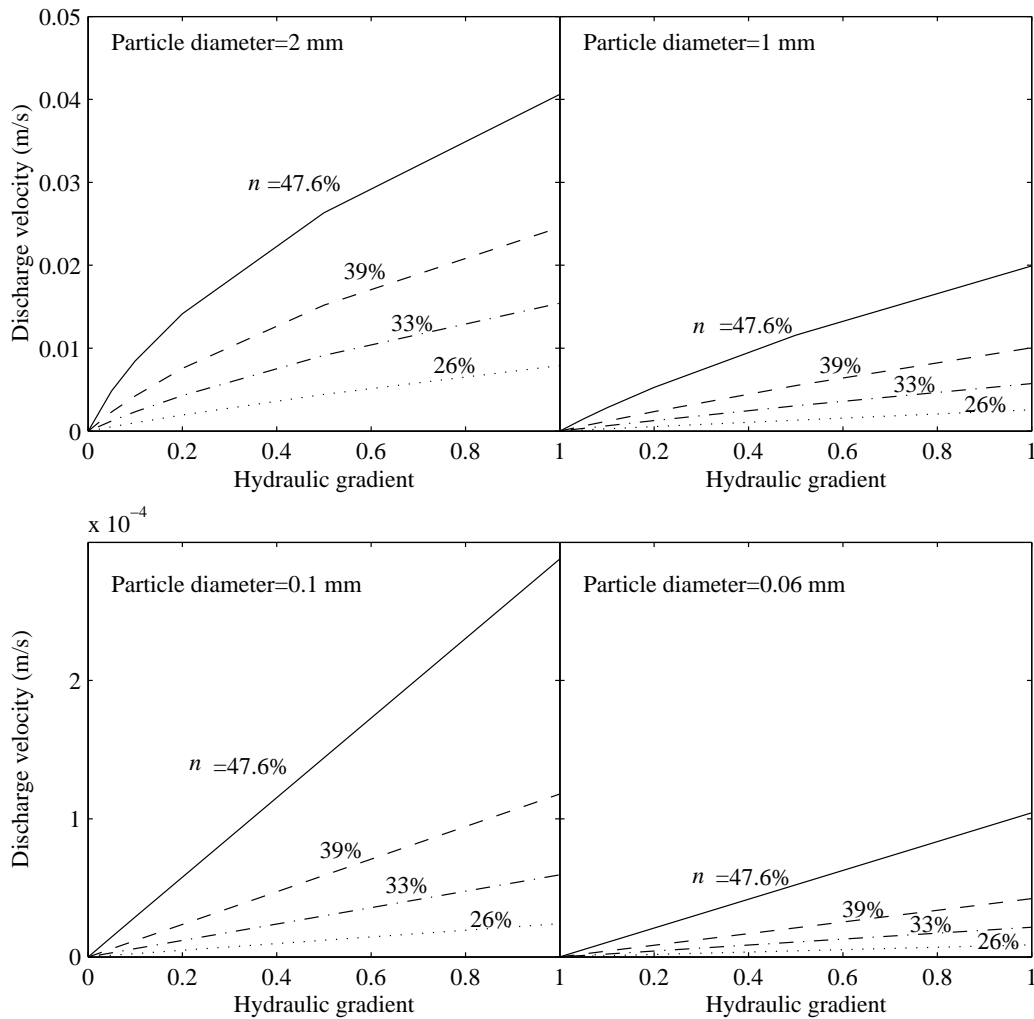
Basic flow problems are used to assess the capabilities of the employed model. Several numerical simulations were performed to investigate the range of validity of Darcy's law. First, the flow of water through a confined soil sample under constant hydraulic gradient is analyzed in a setup similar to laboratory constant-head permeability tests (Fig. 1a). Second, a numerical simulation is conducted on a soil sample with a free surface and subjected to an upward flow (Fig. 1b). In these analyses, water is assumed to have a viscosity of 0.001 Pa.sec, and a density of 1000 kg/m<sup>3</sup>.

### Validity of Darcy's Law

The discrete element code PFC3D is used to generate arrays of spherical uniformly-sized particles having different porosities (ranging between the limiting values of 26.0% and 47.6%, Lambe and Whitman 1969). Figure 2 shows the discharge velocities calculated for a range of hydraulic gradients of practical interest. For relatively small particles sizes (less than 1 mm in diameter) Darcy's law remains valid up to at least a hydraulic gradient of 1. For larger particles (1 mm and 2 mm in diameter in Fig. 2), deviation from Darcy's law is observed for hydraulic gradients less than 1. This deviation becomes more pronounced in high porosity arrays and consistently occurred for Reynolds number ranging between about 1 to 4. This behavior is in agreement with experimental observations of Scheidegger (1974) and Lindquist (1933).

### Upward Flow through a Sand Bed

An upward flow simulation is conducted using a soil sample composed of 10,664 randomly distributed spherical particles with diameters ranging from 0.6 mm to 1.4 mm. The sample is subjected first to a hydrostatic water pressure (to account for buoyancy) and then to an upward hydraulic gradient of 0.1. Figure 3 shows the time history of the average vertical "effective"



**FIG. 2. Variation of discharge velocity with hydraulic gradient for idealized soil samples of different particle sizes**

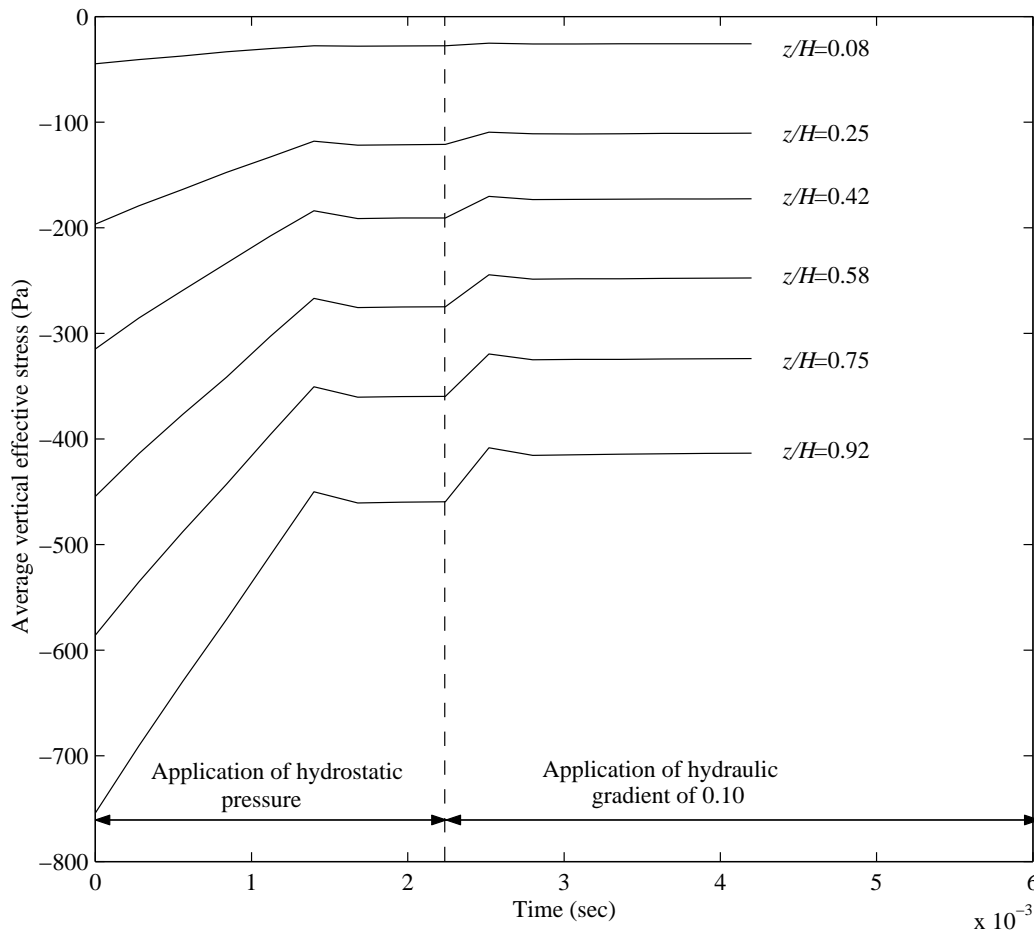
stress evaluated from the inter-particle contact forces. As shown in this figure, and as expected, a significant decrease in vertical stresses is associated with buoyancy and applied upward flow. Analyses of soil response under critical flow conditions are underway and will be published elsewhere.

## CONCLUSION

This paper presented a computational micro-mechanical model for coupled analysis of pore water flow and deformation of granular assemblies. The results of numerical simulations of water seepage through soils are found to be consistent with published experimental observations. Interstitial water flow deviates from Darcy's law when the Reynolds number exceeds a value ranging between about 1 to 4. Research is currently underway to investigate the complex fluid-particle response mechanism under extreme conditions such as liquefaction and piping.

## ACKNOWLEDGEMENT

This research was supported by the National Science Foundation, grant number CMS-



**FIG. 3. Time-history of average vertical “effective” stresses ( $z$  is depth below soil surface and  $H$  is soil sample height).**

0084591. This support is gratefully acknowledged.

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