Trellis material library
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Linear Elasticity

We consider the classical elasticity problem. With the unknown displacements $\boldsymbol{u}$, the stress σ, the body force per unit volume $\boldsymbol{b}$, the prescribed surface traction $\bar{\boldsymbol{t}}$ and the prescribed displacements $\bar{\boldsymbol{u}}$, and the normal vector $\boldsymbol{n}$ we define the governing equations as follows:

$$\begin{aligned} \mathrm{Div}\,\sigma + \rho \boldsymbol{b} &= 0 \quad \text{in } \Omega \\ \boldsymbol{u} &= \bar{\boldsymbol{u}} \quad \text{on } \Gamma_u \\ \sigma \boldsymbol{n} &= \bar{\boldsymbol{t}} \quad \text{on } \Gamma_t \end{aligned} \tag{EQ 1}$$

We choose linear elastic behaviour (Hooke's material law)

$$\sigma = \boldsymbol{E}\varepsilon \tag{EQ 2}$$

with ε being the symmetric gradient $\varepsilon = \nabla^s u$ of the displacements. The elasticity matrix $\boldsymbol{E}$ is given as

$$\boldsymbol{E} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \tag{EQ 3}$$

with the youngs modulus E and the Poisson ratio ν.

To choose this material for the calculation within the SCOREC analysis framework the imageClass of the group "deformable material" has to be set to "linear elastic isotropic

material”. Furthermore, the values for the youngs modulus “E” and the Poisson ratio “nu” have to be specified.

Example:

```
AttCase *case1 = mngr.newCase(“uniaxial tension”,”problem specification”);
ModelAssociation *ModelAss = c1->newModelAssoc();
{
   AttGroup *d = mngr.newGroup(“”,”deformable material”);
   d->imageClass(“linear elastic isotropic material”);
   AttInfoDouble *E = mngr.newDouble(“E”,”E”);
   AttInfoDouble *nu = mngr.newDouble(“nu”,”nu”);
   *E = 210000.0;
   *nu = 0.296;
   d->add(E);
   d->add(E);

   ModelAss->add(d);
   case1->add(d);
 }
```