Trellis material library
Ottmar Klaas

Nonlinear Elasticity

Neo Hooke I

The Neo Hooke material law is fully nonlinear in the displacements and the strains. It can therefore be used for large displacement/large strain calculations.

The free energy function we consider is given as

$$W = \frac{1}{2}\kappa(J-1)^2 - \mu \ln J + \frac{1}{2}\mu(\mathrm{tr}\boldsymbol{C} - 3) \qquad \text{(EQ 1)}$$

with the right Cauchy Green strain tensors $\boldsymbol{C}$, and the material constants κ (bulk modulus) and μ (shear modulus). J represents the determinant of the deformation gradient $J = |\boldsymbol{F}| = |1 + \nabla \boldsymbol{u}|$. The 2nd Piola Kirchhoff stress $\boldsymbol{S}$ tensor can now be derived as

$$\boldsymbol{S} = (\kappa J(J-1) - \mu)\boldsymbol{C}^{-1} + \mu \boldsymbol{1} \qquad \text{(EQ 2)}$$

where $\boldsymbol{1}$ is the second order unit tensor. The elasticity tensor is given as

$$\mathbb{C} = (2\kappa J^2 - \kappa J)\boldsymbol{C}^{-1} \otimes \boldsymbol{C}^{-1} - 2(\kappa J(J-1) - \mu)\mathbb{I}_{C^{-1}} \qquad \text{(EQ 3)}$$

with the fourth order tensor $[\mathbb{I}_{C^{-1}}]^{ijkl} = [\boldsymbol{C}^{-1}]^{ik}[\boldsymbol{C}^{-1}]^{jl}$.

To choose this material for the calculation within the SCOREC analysis framework the imageClass of the group “deformable material” has to be set to “Neo Hooke material I”.

Example:

```
AttCase *case1 = mngr.newCase(“uniaxial tension”,”problem specification”);
ModelAssociation *ModelAss = c1->newModelAssoc();
{
   AttGroup *d = mngr.newGroup(“”,”deformable material”);
   d->imageClass(“Neo Hooke  materia I”);
   AttInfoDouble *kappa = mngr.newDouble(“kappa”,”kappa”);
   AttInfoDouble *mu = mngr.newDouble(“mu”,”mu”);
   *kappa = 7999.47;
```

```
  *mu = 0.8;
  d->add(kappa);
  d->add(mu);

  ModelAss->add(d);
  case1->add(d);
 }
```

Literature

[1] U. Brink, E. Stein [1996]: On some mixed finite element methods for incompressible and nearly incompressible finite elasticity. Comp. Mech. 19, 105 - 119.