

Nonlinear Elasticity

Neo Hooke II

The Neo Hooke material law is fully nonlinear in the displacements and the strains. It can therefore be used for large displacement/large strain calculations.

The free energy function we consider is given as

$$W = \quad \quad \quad (\text{EQ 1})$$

with the right Cauchy Green strain tensors \mathbf{C} , and the material constants κ (bulk modulus) and μ (shear modulus). J represents the determinant of the deformation gradient $J = |\mathbf{F}| = |1 + \nabla \mathbf{u}|$. The Kirchhoff stress $\boldsymbol{\tau}$ tensor can now be derived as

$$\boldsymbol{\tau} = (\kappa J(J - 1) - \mu) \mathbf{I} + \mu \mathbf{b} \quad \quad \quad (\text{EQ 2})$$

where \mathbf{I} is the second order unit tensor. The elasticity tensor is given as

$$\mathbb{C} = (2\kappa J^2 - \kappa J) \mathbf{I} \otimes \mathbf{I} - 2(\kappa J(J - 1) - \mu) \mathbb{I} \quad \quad \quad (\text{EQ 3})$$

with the fourth order unity tensor $[\mathbb{I}]^{ijkl} = [\delta]^{ik} [\delta]^{jl}$.

To choose this material for the calculation within the SCOREC analysis framework the image-Class of the group “deformable material” has to be set to “Neo Hooke material II”.

Example:

```
AttCase *case1 = mngr.newCase("uniaxial tension","problem specification");
ModelAssociation *ModelAss = c1->newModelAssoc();
{
  AttGroup *d = mngr.newGroup("","deformable material");
  d->imageClass("Neo Hooke materia II");
  AttInfoDouble *kappa = mngr.newDouble("kappa","kappa");
  AttInfoDouble *mu = mngr.newDouble("mu","mu");
  *kappa = 7999.47;
```

```
*mu = 0.8;  
d->add(kappa);  
d->add(mu);  
  
ModelAss->add(d);  
case1->add(d);  
}
```

Literature

- [1] U. Brink, E. Stein [1996]: On some mixed finite element methods for incompressible and nearly incompressible finite elasticity. *Comp. Mech.* 19, 105 - 119.