Trellis material library
Ottmar Klaas

Nonlinear Elasticity

Neo Hooke III

The Neo Hooke material law is fully nonlinear in the displacements and the strains. It can therefore be used for large displacement/large strain calculations.

For nearly incompressible material it may be advantageous to split the free energy into a volumetric and isochoric part where the isochoric part is independent of det determinant of the deformation graident. The free energy function we consider for this case is given as

$$W = \frac{1}{2}\kappa(J-1)^2 + \frac{1}{2}\mu(J^{-2/3}\mathrm{tr}\boldsymbol{C} - 3) \qquad \text{(EQ 1)}$$

with the right Cauchy Green strain tensors $\boldsymbol{C}$, and the material constants κ (bulk modulus) and μ (shear modulus). J represents the determinant of the deformation gradient $J = |\boldsymbol{F}| = |1 + \nabla \boldsymbol{u}|$. The 2nd Piola Kirchhoff stress $\boldsymbol{S}$ tensor can now be derived as

$$\boldsymbol{S} = \left(\kappa J(J-1) - \frac{1}{3}\mu J^{-2/3}\mathrm{tr}\boldsymbol{C}\right)\boldsymbol{C}^{-1} + \mu J^{-2/3}\boldsymbol{1} \qquad \text{(EQ 2)}$$

where $\boldsymbol{1}$ is the second order unit tensor. The elasticity tensor is given as

$$\mathbb{C} = \mathbb{C}_{vol} + \mathbb{C}_{iso} \qquad \text{(EQ 3)}$$

where

$$\mathbb{C}_{vol} = (2\kappa J^2 - \kappa J)\boldsymbol{C}^{-1} \otimes \boldsymbol{C}^{-1} - 2\kappa J(J-1)\mathbb{I}_{C^{-1}} \qquad \text{(EQ 4)}$$

and

$$\mathbb{C}_{iso} = \frac{2}{3}\mu J^{-2/3}\left(\frac{1}{3}\mathrm{tr}\boldsymbol{C}\ \boldsymbol{C}^{-1} \otimes \boldsymbol{C}^{-1} - \boldsymbol{C}^{-1} \otimes \boldsymbol{1} - \boldsymbol{1} \otimes \boldsymbol{C}^{-1} + \mathrm{tr}\boldsymbol{C}\mathbb{I}_{C^{-1}}\right) \qquad \text{(EQ 5)}$$

with the fourth order tensor $[\mathbb{I}_{C^{-1}}]^{ijkl} = [\boldsymbol{C}^{-1}]^{ik}[\boldsymbol{C}^{-1}]^{jl}$.

To choose this material for the calculation within the SCOREC analysis framework the image-Class of the group “deformable material” has to be set to “Neo Hooke material III”.

Example:

```
AttCase *case1 = mngr.newCase(“uniaxial tension”,”problem specification”);
ModelAssociation *ModelAss = c1->newModelAssoc();
{
   AttGroup *d = mngr.newGroup(““,”deformable material”);
   d->imageClass(“Neo Hooke  materia III”);
   AttInfoDouble *kappa = mngr.newDouble(“kappa”,”kappa”);
   AttInfoDouble *mu = mngr.newDouble(“mu”,”mu”);
   *kappa = 7999.47;
   *mu = 0.8;
   d->add(kappa);
   d->add(mu);

   ModelAss->add(d);
   case1->add(d);
 }
```

Literature

[1] U. Brink, E. Stein [1996]: On some mixed finite element methods for incompressible and nearly incompressible finite elasticity. Comp. Mech. 19, 105 - 119.