Trellis material library
Ottmar Klaas

Nonlinear Elasticity

St. Venant-Kirchhoff Material

The St. Venant-Kirchhoff Material is fully nonlinear in the displacements. It can therefore be used for large displacement calculations where the material undergoes only small strains.

The free energy function we consider

$$W(\boldsymbol{E}) = \frac{\lambda}{2}(\mathrm{tr}\boldsymbol{E})^2 + \mu \mathrm{tr}\boldsymbol{E}^2 \qquad \text{(EQ 1)}$$

with the Green strain tensors $\boldsymbol{E}$, and the Lame constants λ and μ. The 2nd Piola Kirchhoff stress $\boldsymbol{S}$ tensor can now be derived as

$$\boldsymbol{S}(\boldsymbol{E}) = \lambda(\mathrm{tr}\boldsymbol{E})\boldsymbol{1} + 2\mu\boldsymbol{E} \qquad \text{(EQ 2)}$$

where $\boldsymbol{1}$ is the second order unit tensor. The elasticity tensor is given as

$$\mathbb{C} = \lambda(\boldsymbol{1} \otimes \boldsymbol{1}) + 2u\mathbb{I} \qquad \text{(EQ 3)}$$

with the fourth order unit tensor $\mathbb{I}$.

To choose this material for the calculation within the SCOREC analysis framework the image-Class of the group “deformable material” has to be set to “St. Venant-Kirchhoff material”. Furthermore, the values for the Lame constants “lambda” and “mu” have to be specified.

Example:

```
AttCase *case1 = mngr.newCase(“uniaxial tension”,”problem specification”);
ModelAssociation *ModelAss = c1->newModelAssoc();
{
   AttGroup *d = mngr.newGroup(“”,”deformable material”);
   d->imageClass(“St. Venant-Kirchhoff material”);
   AttInfoDouble *lambda = mngr.newDouble(“lambda”,”lambda”);
```

```
  AttInfoDouble *mu = mngr.newDouble(“mu”,”mu”);
  *lambda = 117556.28;
  *mu = 81018.52;
  d->add(lambda);
  d->add(mu);

  ModelAss->add(d);
  case1->add(d);
}
```