```
> restart; with(linalg):
```

Warning, the protected names norm and trace have been redefined and unprotected

$$clck := 0.5026548246$$

Problem 1) given the velocity field

```
>
```

```
> phi:=3/2*x^2*y^2-1/3*(x^3+sin(3*x));u:=diff(phi,x);v:=diff(phi,y);
```

$$\phi := \frac{3}{2}x^2y^2 - \frac{1}{3}x^3 - \frac{1}{3}\sin(3\,x)$$

$$u := 3\,x\,y^2 - x^2 - \cos(3\,x)$$

$$v := 3\,x^2\,y$$

Above we found the u =partial phi/partial x and v =partial phi/partial y (5 points each 3 for formula 2 for correct math).

to do part b we could dive in or take the "hint" "if it exists" and first check if this velocity is divergence free (i.e. div(V)=0)

When we check it below we see that it is not. If you did that and stated that psi cannot exist if continuity is not satisfied then those are the quickest 10 points known to man. Breakdown this way is 5 points for knowing that divV=0 implies that the stream function does not exist. THEN writing div V correctly (expanding it out) =3 points and then 2 pts of the correct partial differentation

```
> divV:=diff(u,x)+diff(v,y);
```

$$divV := 3\,y^2 - 2\,x + 3\sin(3\,x) + 3\,x^2$$

If you did not think to check continuity then the brute force way by integrating the velocity field from the definition of u and v from psi yields

NOTE you get points from one approach or the other NOT BOTH. We tried to grade the approach that gave you the best score.

```
> psi_u:=int(u,y)+fxpsi; psi_v:=int(-1*v,x)+fypsi; diff=psi_u-psi_v;
```

$$psi\_u := x\,y^3 - x^2\,y - \cos(3\,x)\,y + fxpsi$$

$$psi\_v := -x^3\,y + fypsi$$

$$diff = x\,y^3 - x^2\,y - \cos(3\,x)\,y + fxpsi + x^3\,y - fypsi$$

From this we see that there is no way to reconcile this (if this worked the diff variable would have had pure functions of x, pure functions of y which would have defined the fxpsi and fypsi functions...as it stands those functions would need to be mixed functions of x and y to make diff=0). If you did it the long way then count 2 points each for correct integral setups, 2 points each for correct integration and 2 pts for showing irreconcilable.

Part c is easy if you note that when a potential exists then line integral is path independant. Rather than choose the difficult path,integrate from x=0..1 while y=0 and then from y=0..1 while x=1 which in maple looks like

```
> int(subs(y=0,u),x=0..1)+int(subs(x=1,v),y=0..1); evalf(%);
```

$$\frac{7}{6} - \frac{1}{3}\sin(3)$$

$$1.119626664$$

Of course there are other more difficult ways of doing this but I won't discuss them here. Setting up the integrals correctly will get you 2 points, evaluating them correctly gives you 2 more, last point is for knowing that you can choose your path when flow is irrotational.

Problem 2.

given the psi, part a just requires us to differentiate psi appropriately to find the velocity components Vr and Vtheta

```
> psi:=Vinf*r*sin(theta)*(1-R^2/r^2)+Gamma*ln(r)/2/Pi;Vr:=diff(psi,theta)/r;
  Vtheta:=-diff(psi,r);  simplify(subs(Gamma=0,%));
```

$$\psi := Vinf\, r \sin(\theta)\left(1-\frac{R^2}{r^2}\right)+\frac{\Gamma \ln(r)}{2\,\pi}$$

$$Vr := Vinf \cos(\theta)\left(1-\frac{R^2}{r^2}\right)$$

$$Vtheta := -Vinf \sin(\theta)\left(1-\frac{R^2}{r^2}\right)-\frac{2\,Vinf \sin(\theta)\,R^2}{r^2}-\frac{\Gamma}{2\,r\,\pi}$$

$$-\frac{Vinf \sin(\theta)\,(r^2+R^2)}{r^2}$$

Again formulas for the correct differentials for each component are worth 3, evaluating them correctly is 2 each.

for part b

We are going to be evaluating -V dot ds around a circle of radius 2R. We have V (the velocity vector above) we need ds. It is obviously easiest to do this integral in polar coordinates so ds=rdtheta etheta where etheta is the unit vector in the theta direction. Since we are going to take the dot product ov ds and V it is clear that only the Vtheta component is going to be relevant and thus our integrand is

-Vtheta*r*dtheta. The last step is to be sure that we get the limits correct. I hope it is obvious that to go around the circle requires theta to go from 0 to 2Pi.

```
> ptb:=int(-Vtheta*r,theta=0..2*Pi);
```

$$ptb := \Gamma$$

Which initially troubles us until we remember that it is only the vortex solution that has any circulation and it is the same at any radius. You get 2 points for drawing your figure correctly, labeling ds. 2 points for getting ds correct. 2 points for correct integrand, 2 pts for correct limit and 2 points for correct final answer

ptc: L'= rho V_inf *Gamma 5 pts

Prob 3

point breakout

1) dz/dx (3)
2) A0 (3)
3) A1 (3)
4) A2 (3)
5) Cl @5deg (4)
6) Cm_le (3)
7) xcp (general or at 5 degrees) (3)
8) alpha L=zero (3)

```
> restart;z:=c*xi*(1-xi)*(2-xi)/10;
  dzdx_xi:=simplify(diff(z,xi)/c);
  maxminroots:=evalf(solve(dzdx_xi=0,xi));
  dzdx:=expand(subs(xi=(1-cos(theta))/2,dzdx_xi));
  maxcamb:=subs(xi=maxminroots[2],z);
  A0:=alpha-1/Pi*int(dzdx,theta=0..Pi);
  A1:=2/Pi*(int(dzdx*cos(theta),theta=0..Pi));
  A2:=2/Pi*(int(dzdx*cos(2*theta),theta=0..Pi));
  cl:=simplify(2*Pi*A0+Pi*A1);
  cmLE:=-Pi/2*(A0+A1 -A2/2);
  cl:=2*Pi*(alpha+1/Pi*int(dzdx*(cos(theta)-1),theta=0..Pi));
  cl_5:=evalf(subs(alpha=5/180*Pi,cl));xcpoc:=1/4-(A2-A1)*Pi/4/cl_5;
  evalf(%); cl2:=2*Pi*(alpha-azero); azero_deg:=solve(cl2=cl,
  azero)*180/Pi; evalf(%); evalf((alpha-A0-A1/2)*180/Pi);
  alztest:=-1/Pi*int(dzdx*(cos(theta)-1),theta=0..Pi); evalf(%*180/Pi);
>
```

$$z := \frac{1}{10} c\, \xi\, (1 - \xi)\, (2 - \xi)$$

$$dzdx\_xi := \frac{1}{5} - \frac{3}{5} \xi + \frac{3}{10} \xi^2$$

$$maxminroots := 1.577350269, 0.4226497307$$

$$dzdx := -\frac{1}{40} + \frac{3}{20} \cos(\theta) + \frac{3}{40} \cos(\theta)^2$$

$$maxcamb := 0.03849001793\, c$$

$$A0 := \alpha - \frac{1}{80}$$

$$A1 := \frac{3}{20}$$

$$A2 := \frac{3}{80}$$

$$cl := 2\, \pi\, \alpha + \frac{1}{8} \pi$$

$$cmLE := -\frac{1}{2} \pi \left(\alpha + \frac{19}{160} \right)$$

$$cl := 2\,\pi\left(\alpha + \frac{1}{16}\right)$$

$$cl\_5 := 0.9410104374$$

$$xcpoc := \frac{1}{4} + 0.02988808508\,\pi$$

$$0.3438961885$$

$$cl2 := 2\,\pi\,(\alpha - azero)$$

$$azero\_deg := -\frac{45}{4\,\pi}$$

$$-3.580986219$$

$$-3.580986219$$

$$alztest := \frac{-1}{16}$$

$$-3.580986219$$

>