1. A 25 ft span wing with straight taper from 6 ft root chord to 3 ft tip chord. The zero-lift angle of the airfoils is -2.0^0 at the root but the camber is adjusted to linearly decrease to -0.5 degrees at the tip. The geometric angle of attack is 7^0 at the root varying linearly to 4^0 at the tips. Determine the lift, the induced drag, and the induced drag efficiency, e, at a sea level airspeed of 230 mph, using the number of terms in the circulation series and control point θ values indicated. Compare the relative errors for lift and drag respectively, assuming for this purpose that the results in part (c) are accurate enough to be considered exact. You must do part a) completely by hand showing all detail. I would suggest checking your results against an application of my Maple notebook. Then if you like you can proceed to do parts b and c using the Maple notebook

Known: b = 25 ft, $V_\infty = 230\,\text{mph}$

$$c(y) = 6 - 3\left(\frac{y}{b/2}\right),\ \alpha_{L=0}(y) = -2.0 + 1.5\left(\frac{y}{b/2}\right),\ \alpha(y) = 7 - 3\left(\frac{y}{b/2}\right)$$

Unknown: c_L, L, c_{Di}, D, e

Equations:

$$\alpha(\theta_0) = \frac{2b}{\pi c(\theta_0)}\sum_1^N A_n \sin n\theta_0 + \alpha_{L=0}(\theta_0) + \sum_1^N nA_n \frac{\sin n\theta_0}{\sin\theta_0}$$

$$\text{AR} = \frac{b^2}{s},\ c_L = \pi \text{AR} A_1,\ L = q_\infty s c_L,\ \delta = \sum_2^N n\left(\frac{A_n}{A_1}\right)^2,\ c_{D,i} = \frac{c_L^2}{\pi\text{AR}}(1+\delta),$$

$$D_i = q_\infty s c_{D,i},\ e = \frac{1}{1+\delta}$$

For elliptical distribution:

$$\Gamma_0 \sin\theta = 2bV_\infty \sum_1^N A_n \sin n\theta$$

$$c_{L,e} = \frac{\Gamma_0 b\pi}{2s},\ c_{D,e} = \frac{c_{L,e}^2}{\pi\text{AR}}$$

Solution: Using Maple: s = 112.5 ft^2, AR = 5.556

a) A_1 =0.03442, A_3 = -0.005548

δ = 0.07795, q_∞ = 135.3 $\frac{lb_f}{ft^2}$

c_L = 0.6007, $c_{D,I}$ = 0.02229

L = 9144 lb_f, D = 339.3 lb_f, , e = 0.9277

b) $A_1 = 0.03319$, $A_3 = -0.003488$, $A_5 = 0.002111$

$\delta = 0.05337$, $q_\infty = 135.3\ \frac{lb_f}{ft^2}$

$c_L = 0.5792$, $c_{D,I} = 0.02025$

$L = 8816\ lb_f$, $D = 308.2\ lb_f$, , $e = 0.9493$

c) $A_1 = 0.03276$, $A_3 = -0.00301$, $A_5 = 0.001869$, $A_7 = -0.0006313$

$\delta = 0.04419$, $q_\infty = 135.3\ \frac{lb_f}{ft^2}$

$c_L = 0.5718$, $c_{D,I} = 0.01956$

$L = 8704\ lb_f$, $D = 297.8\ lb_f$, , $e = 0.9577$

d) Using $\theta = \frac{\pi}{2}$ the coefficients from part c: $\frac{\Gamma_0}{V_\infty} = 1.914$

$c_L = 0.668$, $c_{D,I} = 0.02557$

$L = 10168\ lb_f$, $D = 389.2\ lb_f$,

Relative errors assuming solution in part c is accurate enough to be considered exact

	% error in L	% error in D_i
2 terms	5.06%	13.94%
3 terms	1.29%	3.49%
Elliptical	16.82%	30.69%

2. A 50 ft. span wing with constant 3 ft chord over the inboard 30 ft, followed by straight taper to 1 ft at the tips. The zero-lift angle of attack is -2.5° throughout, and the geometric angle of attack is 7° at the root, with linear washout to 4° at the tips. The wing loading is 10 lbs/ft^2.

(a) Determine the wing lift coefficient, the sea level airspeed, the induced drag coefficient, and the induced drag correction factor, using your idea of a reasonable number of terms in the circulation series.

(b) Recalculate the results with the geometric angle of attack reduced by 3°. You will probably find that this corresponds to a significant increase in airspeed. Comment on the use of washout in the light of your part (a) and (b) results for the induced drag correction factor and induced drag coefficient.

(c) Repeat (b) assuming an elliptic distribution (use the Γ_o you computed already from the series solution.)

Known: b = 50 ft, $w = 10$ lbs/ft^2, $\alpha_{L=0} = -2.5$

$$c(y) = \begin{cases} 3 & |y| < 15 \\ 6 - \frac{1}{5}y & |y| > 15 \end{cases}, \quad \alpha(y) = 7 + 3\left(\frac{y}{b/2}\right)$$

Unknown: c_L, V_∞, c_{Di}, e

Equations:

$$\alpha(\theta_0) = \frac{2b}{\pi c(\theta_0)} \sum_1^N A_n \sin n\theta_0 + \alpha_{L=0}(\theta_0) + \sum_1^N nA_n \frac{\sin n\theta_0}{\sin\theta_0}$$

$$\text{AR} = \frac{b^2}{s}, \quad c_L = \pi \text{AR} A_1, \quad \delta = \sum_2^N n\left(\frac{A_n}{A_1}\right)^2, \quad c_{D,i} = \frac{c_L^2}{\pi \text{AR}}(1+\delta), \quad e = \frac{1}{1+\delta}$$

$$W = \frac{1}{2}\rho_\infty V_\infty^2 c_L$$

For elliptical distribution:

$$\Gamma_0 \sin\theta = 2bV_\infty \sum_1^N A_n \sin n\theta$$

$$c_{L,e} = \frac{\Gamma_0 b\pi}{2s}, \quad c_{D,e} = \frac{c_{L,e}^2}{\pi \text{AR}}$$

Solution: Using Maple: s = 130 ft^2, AR = 19.23

a) A_1 =0.01352, A_3 = -0.0007988, A_5 = -0.0001313, δ = 0.01094

e = 0.9892, V_∞ = 101.5 ft/s = 69.18 mph

$c_L = 0.8169$, $c_{D,I} = 0.01117$

b) $A_1 = 0.008624$, $A_3 = -0.0009307$, $A_5 = 0.0000421$, $\delta = 0.03506$
$e = 0.9661$, $V_\infty = 127.1$ ft/s = 86.63 mph

$c_L = 0.521$, $c_{D,I} = 0.00465$

c) $\theta = \dfrac{\pi}{2}$ the coefficients from part c: $\dfrac{\Gamma_0}{V_\infty} = 0.9596$

$c_L = 0.5298$, $c_{D,I} = 0.005564$, $V_\infty = 120.4$ ft/s = 82.12 mph