Aerodynamics I : MANE 4070

Homework #1

1.4

- Problem statement

Consider an infinitely thin flat plate with a 1 m chord c at an angle of attack of 10° in a supersonic flow. The pressure and shear stress distributions on the upper and lower surfaces are given by:

$p_u = 4\times10^4(x-1)^2 + 5.4\times10^4 \; [N/m^2]$

$p_\ell = 2\times10^4(x-1)^2 + 1.73\times10^5 \; [N/m^2]$

$\tau_u = 288\,x^{-0.2} \; [N/m^2]$

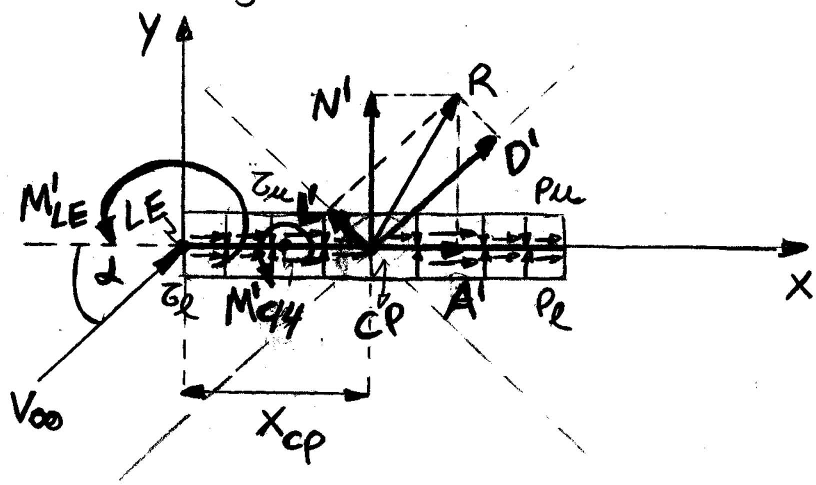
$\tau_\ell = 731\,x^{-0.2} \; [N/m^2]$

respectively, where x is the distance from the leading edge in meters and p and τ are in newtons per square meter. Calculate the normal and axial forces, the lift and drag, moments about the leading edge, and moments about the quarter chord, all per unit span. Also, calculate the location of the center of pressure.

- Known: supersonic flow, chord, angle of attack, p_u, p_ℓ, τ_u, τ_ℓ

- Unknown: Normal and Axial forces $\to N', A'$
  Lift and Drag $\to L', D'$
  Moment about leading edge $\to M'_{LE}$
  Moment about quarter chord $\to M'_{c/4}$
  Location of the center of pressure $\to x_{cp}$

- Diagram.



- Equations

$$N' = \int_0^c (p_l - p_u)\,dx$$
$$A' = \int_0^c (\tau_l - \tau_u)\,dx$$

For a flat plate $\theta = 0$

$$L' = N'\cos\alpha - A'\sin\alpha$$
$$D' = N'\sin\alpha + A'\cos\alpha$$

$$M'_{LE} = \int_0^c (p_u - p_l)\,x\,dx$$
$$M'_{c/4} = M'_{LE} + L'(c/4)$$

For a flat plate $\theta = 0$

- Answer

$$N' = \int_0^1 \left(-2\times10^4 (x-1)^2 + 1.19\times10^5\right) dx = -2\times10^4 \left(\frac{x^3}{3} - x^2 + x\right)_0^1 +$$
$$\left(1.19\times10^5 x\right)_0^1 = \boxed{1.12\times10^5\ N}$$

$$A' = \int_0^1 \left(731 x^{-0.2} + 288 x^{-0.2}\right) dx = \left(1274 x^{0.8}\right)_0^1 = \boxed{1274\ N}$$

$$L' = 1.12\times10^5 \cos 10^\circ - 1274 \sin 10^\circ = \boxed{1.105\times10^5 N}$$

$$D' = N'\sin\alpha + A'\cos\alpha = 1.12\times10^5 \sin 10^\circ + 1274 \cos 10^\circ$$

$$\boxed{D' = 2.07\times10^4 N}$$

$$M'_{LE} = \int_0^1 \left(2\times10^4(x-1)^2 - 1.19\times10^5\right) x\,dx + 2\times10^4\left(\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2}\right)_0^1 - \left(0.595\times10^5 x^2\right)_0^1 = \boxed{-5.78\times10^4 Nm}$$

$$M'_{c/4} = -5.78\times10^4 + 1.105\times10^5(0.25) = \boxed{-3.02\times10^4 Nm}$$

$$x_{cp} = -\frac{M'_{LE}}{N'} = -\frac{(-5.78\times10^4)}{1.12\times10^5} = \boxed{0.516 m}$$

2.4

- Problem statement: Consider a velocity field where the x and y components of velocity are given by:
- Known $\rightarrow u = cy/(x^2+y^2)$ and $v = -cx/(x^2+y^2)$ where c is a constant
- Unknown: obtain the equations of the streamlines $\psi = f(x,y)$
- Equations $\rightarrow \dfrac{dx}{u} = \dfrac{dy}{v}$ 'or' $\dfrac{dy}{dx} = \dfrac{v}{u}$
- Answer

$$\frac{dy}{dx} = \frac{v}{u} = \frac{-\cancel{c}x\,\cancel{(x^2+y^2)}}{\cancel{c}y\,\cancel{(x^2+y^2)}} = -\frac{x}{y}$$

$$y\,dy = -x\,dx$$

Integrating in both sides;

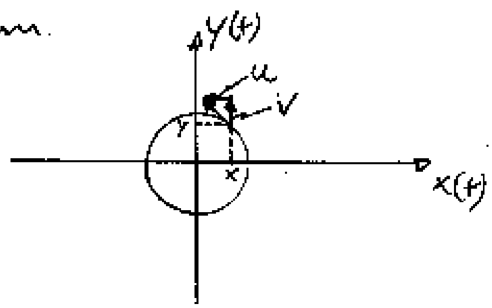
$$\frac{y^2}{2} = -\frac{x^2}{2} + C_1' \quad \text{'or'} \quad y^2 = -x^2 + \underbrace{2C_1'}_{\equiv C}$$

Finally,

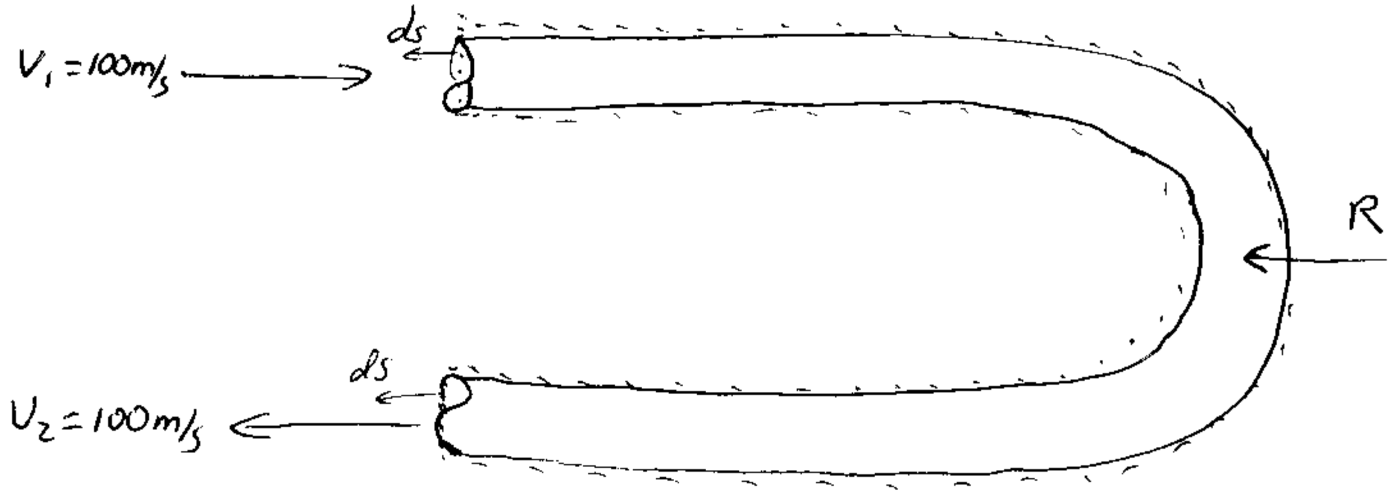
$$\boxed{x^2+y^2 = C_1 = \text{constant}}$$

The streamlines are concentric circumferences withe their center at the origin.

- Diagram.



2.12 Consider a length of pipe bent into a U-shape. The inside diameter of the pipe is .5m. Air enters one leg of the pipe @ a mean velocity of 100 m/s and exits the other leg at the same magnitude of velocity, but moving in the opposite direction. The pressure of the flow at the inlet and exit is the ambient pressure of the surroundings. Calculate the magnitude and direction of the force exerted on the pipe by the airflow. $\rho_{air} = 1.23 \frac{kg}{m^3}$



momentum eqn:

for constant pressure → 0   neglect body forces   neglect viscous forces

$$\cancel{\frac{d}{dt}\iiint_V \rho \vec{V} dv} + \oiint_S (\rho \vec{V} \cdot ds)\vec{V} = -\cancel{\oiint_S p\, ds} - \cancel{\iiint_V \rho f dv} + \cancel{F_{viscous}} - \bar{R}$$

0 since steady

momentum eqn becomes   and $V_1 = -V_2$

$$-\rho V_1 A V_1 + -\rho(V_2)(A)(V_2) = -R$$

$$-\rho V_1^2 A - \rho V_1^2 A = R = -2\rho V_1^2 A = 2(1.23)(100)^2 \frac{\pi(.5)^2}{4} = 4830N$$