<u>9.3</u>

Eqn. 8.80 does not hold for an oblique shock wave, and hence the column in Appendix B labeled $\frac{p_{0,2}}{p_1}$ *cannot* be used, in conjunction with the normal component of the upstream Mach number, to obtain the total pressure behind an oblique shock wave. On the other hand, the column labeled $\frac{p_{0,2}}{p_{0,1}}$ can be used for an oblique shock, using $M_{n,1}$. Explain why this is so.

Soln.

Changes across an oblique shock wave are governed only by the component of velocity normal to the wave. To derive Eqn. 8.80, the following eqns are used.

$$\frac{p_{0,2}}{p_2} = \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\frac{\gamma}{\gamma - 1}} \tag{1}$$

$$M_2^2 = \frac{1 + \left(\frac{\gamma - 1}{2}\right) M_1^2}{\gamma M_1^2 - \frac{\gamma - 1}{2}} \tag{2}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_1^2 - 1\right) \tag{3}$$

For an oblique shock, the normal components of the Mach number are used across the shock, in Eqns (2) and (3). However, the total Mach number is used for the isentropic relationship in Eqn. (1). Therefore for an oblique shock, Eqn. 8.80 cannot be used.

The relationship between the total pressures across a normal shock is based on the entropy difference between two states for a calorically perfect gas. This relationship still applies across an oblique shock using the normal component of the Mach numbers.

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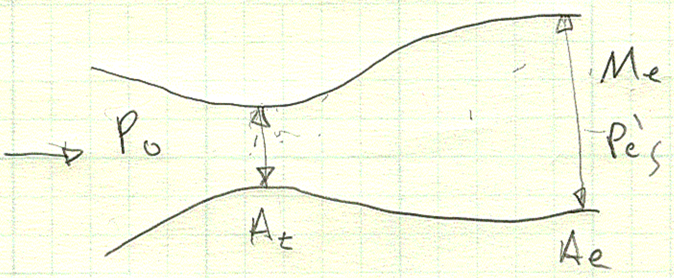
10.9 Consider a convergent-divergent nozzle with a exit to throat area ratio of 1.53. The reservoir pressure is 1 atm. Assuming isentropic flow, except for the possibility of a normal shock wave inside the nozzle, calculate the exit Mach number when the exit pressure

P_e is a) 0.94 atm b) 0.886 atm c) 0.75 atm d) 0.154 atm

Known: $\frac{A_e}{A_t} = 1.53$, P_e's

Unknown: M_e @ P_e's



a) $P_o = 1$ atm, $\frac{A_e}{A_t} = 1.53$

$P_e = 0.94$ atm

$$\frac{P_o}{P_e} = \frac{1\text{ atm}}{0.94\text{ atm}} = 1.064$$

$$\therefore \quad 1.064 = (1 + 0.2 M_e^2)^{3.5}$$

Maple $\boxed{M_e = 0.298}$

$$\left(\frac{A_e}{A^*}\right)^2 = \frac{1}{M_e^2}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} M_e^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$

Maple $\frac{A_e}{A^*} = 2.043$

$$\frac{A_t}{A^*} = \left(\frac{A_e}{A_t}\right)^{-1}\left(\frac{A_e}{A^*}\right)$$

$$\frac{A_t}{A^*} = 1.33$$

$\therefore \quad A_t = 1.33\,A^* \quad \equiv \quad A_t > A^*$

Since A_t is greater than A^* the flow is not "Sonic" at the throat and is Subsonic everywhere.

Part b) $P_o = 1\,atm \qquad \frac{A_e}{A_t} = 1.53 \qquad P_e = 0.886\,atm$

$$\frac{P_o}{P_e} = \frac{1}{0.886} = 1.128$$

MAple $\qquad M_e = 0.419$

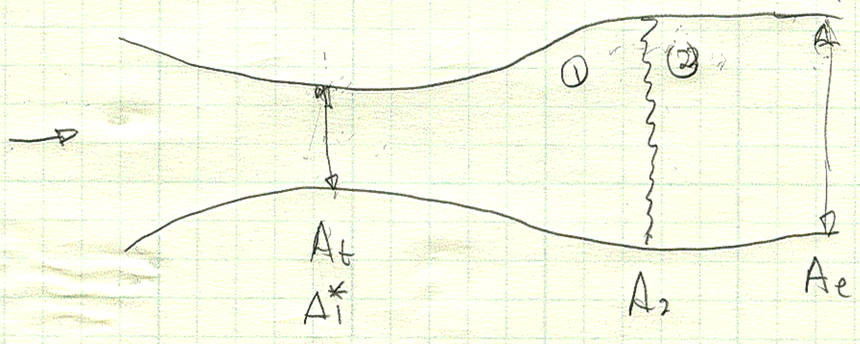
MAple $\qquad \frac{A_e}{A^*} = 1.530$

$$\frac{A_t}{A^*} = 1.000$$

$\therefore \quad A_t = (1.0)\,A^*$

$\therefore$ Since $A_t = A^*$ the flow is "Sonic" at the throat and Subsonic elsewhere

Part c)



From previous work we need to guess between 1 for sonic and actual nozzle ratio or 1.53.

Try any - perhaps $\frac{A_2}{A_1^*} = 1.204$

MAple

$$\boxed{\begin{aligned} M_e &= 0.469 \\ P_e &= 0.788 \end{aligned}}$$

$P_e \neq 0.75$

try $\frac{A_2}{A_1^*} = 1.301$

$M_e = 0.5029$

$P_e' = 0.7336$

"Pe too low"

interpolate

$$\frac{x_m - x_1}{x_2 - x_1} = \frac{y_m - y_1}{y_2 - y_1}$$

$$y_m = y_1 + (y_2 - y_1)\left(\frac{x_m - x_1}{x_2 - x_1}\right)$$

$$\frac{A_2}{A_1^*} = 1.27$$

MAple

$$\boxed{\begin{aligned} M_e &= 0.492 \\ P_e &= 0.7499 \end{aligned}}$$ ✓

Part d) $\frac{P_o}{P_e} = 6.493$

Maple $M_e = 1.879$

$\frac{A_e}{A^*} = 1.53$ = exactly the nozzle ratio ✓

$A_t = (1.0)\,A^*$ Sonic at throat

∴ Sonic at throat, isentropic expansion with $M_e = 1.88$ ✓