

U.2 Using the Prandtl-Glauert rule, calculate the lift coefficient for an NACA 2412 airfoil at $\alpha = 5^\circ$ in a Mach 0.6 free stream.

Known: $\alpha = 5^\circ$, $M = 0.6$

Unknown: C_L

Fig 4.5 @ $\alpha = 5^\circ$

$$C_{L_0} \approx 0.75$$

$$C_L = \frac{C_{L_0}}{\sqrt{1 - M^2 \sin^2 \alpha}}$$

$$C_L = 0.9375$$

11.4

In low-speed incompressible flow, the peak pressure coefficient at (at minimum pressure point) on an airfoil is -0.41 . Estimate the critical Mach number for this airfoil using the Prandtl-Glauert rule.

Known: $C_{p,\min} = -0.41$

Maple analytical solution

$$M_{cr} = 0.743$$

12.1 Consider $\alpha = 5^\circ = 0.0873$ rad.

$$c_t = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \frac{4(0.0873)}{\sqrt{(2.6)^2 - 1}} = 0.1455$$

From exact theory (Prob. 9.13): $c_t = 0.148$

$$\% \text{ error} = \frac{0.148 - 0.1455}{0.148} \times 100 = 1.69\%$$

$$c_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} = c_t \alpha = (0.1455)(0.0873) = 0.0127$$

From exact theory (Prob. 9.13): $c_d = 0.0129$

$$\% \text{ error} = \frac{0.0129 - 0.0127}{0.0129} \times 100 = 1.53\%$$

(b) $\alpha = 15^\circ = 0.2618$ rad

$$c_t = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = 0.436$$

From exact theory (Prob. 9.13): $c_t = 0.452$

$$\% \text{ error} = \frac{0.452 - 0.426}{0.452} \times 100 = 3.47\%$$

$$c_d = c_t \alpha = (0.436)(0.2618) = 0.114$$

From exact theory (Prob. 9.13): $c_d = 0.121$

$$\% \text{ error} = \frac{0.121 - 0.114}{0.121} \times 100 = 5.7\%$$

(c) $\alpha = 30^\circ = 0.5236$ rad

$$c_t = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \frac{4(0.5236)}{\sqrt{(2.6)^2 - 1}} = 0.873$$

From exact theory (Prob. 9.13): $c_t = 1.19$

$$\% \text{ error} = \frac{1.19 - 0.873}{1.19} \times 100 = 26.7\%$$

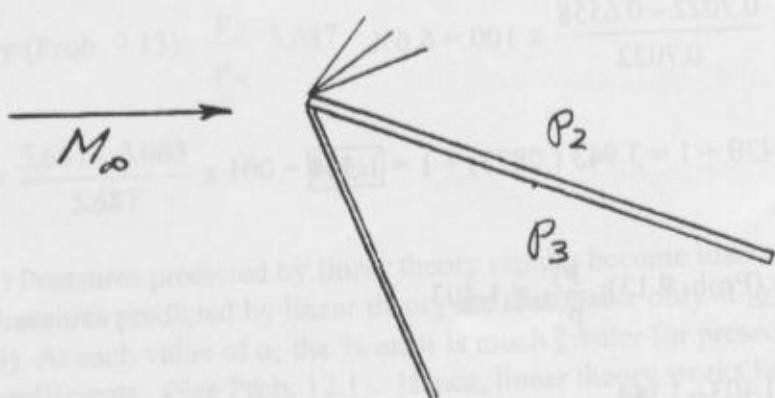
$$c_d = c_t \alpha = (0.873)(0.5236) = 0.457$$

From exact theory (Prob. 9.13): $c_d = 0.687$

$$\% \text{ error} = \frac{0.687 - 0.457}{0.687} = 33.5\%$$

Conclusion: At low α , linear theory is reasonably accurate. However, its accuracy deteriorates rapidly at high α . This is no surprise; we do not expect linear theory to hold for large perturbations. It appears that linear theory is reasonable to at least 5° , and that it is acceptable as high as 15° . At 30° it is unacceptable. Keep in mind that the above comments pertain to the lift and wave drag coefficients only. They say nothing about the accuracy of the pressure distributions themselves.

12.2



Conclusions: (1) Pressures predicted by linear theory increase as α increases. (2) Pressures predicted by linear theory are reasonable for small values of α ($< 5^\circ$) but less accurate for larger values of α . (3) Linear theory is better for lift and wave drag coefficients than for pressure distributions. (4) The lift coefficient goes to zero as α approaches 90° . What happens is that the angle between the free-stream velocity and the chord vector goes to zero. The lift coefficient goes to zero because the chord vector has no component in the direction of the free-stream velocity.

not correct & long

$$C_e = \frac{q_2}{\frac{1}{T}} = \frac{q(15\pi/180)}{\cancel{q^2} - \cancel{1}} = \frac{q(15\pi/180)}{4\pi(\alpha^2 + \Delta\theta^2)}$$

$$C_d = \frac{1}{\frac{1}{T}} \left(\left(\frac{15\pi}{180} \right)^2 + \left(\frac{\Delta\theta}{180} \right)^2 \right)$$

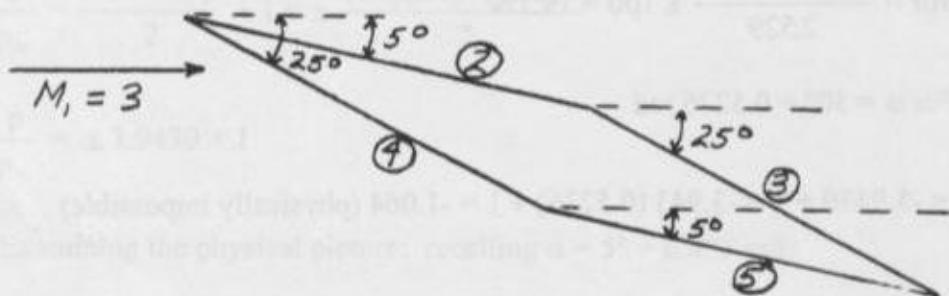
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12.3 $\frac{p}{p_\infty} = \frac{\gamma M_\infty^2 C_p}{2} + 1$ where $C_p = \pm \frac{2\theta}{\sqrt{M_\infty^2 - 1}}$

$$C_p = \pm \frac{2\theta}{\sqrt{(3)^2 - 1}} = \pm 0.70710$$

$$\frac{p}{p_\infty} = \pm \frac{(1.4)(3)^2 (0.7071)\theta}{2} + 1$$

$$\frac{p}{p_\infty} = \pm 4.4550 + 1$$



Surface 2: $\theta = 5^\circ = 0.08727$ rad.

$$\frac{p_2}{p_\infty} = -4.455 (.08727) + 1 = 0.6112$$

Surface 3: $\theta = 25^\circ = 0.4363$ rad

$$\frac{p_3}{p_\infty} = -4.455 (.4363) + 1 = -0.9439$$

Surface 4: $\theta = 25^\circ = 0.4363$ rad

$$\frac{p_4}{p_\infty} = 4.455 (.4363) + 1 = 2.944$$

Note: Although a negative pressure is not physically possible, in order to calculate the net force, we must carry it as such.

Surface 5: $\theta = 5^\circ = 0.08727 \text{ rad}$

$$\frac{p_5}{p_\infty} = 4.455 (.08727) + 1 = 1.3888$$

$$c_t = \frac{2}{\gamma M_1^2} \frac{\ell}{c} \left[\left(\frac{p_4}{p_\infty} - \frac{p_3}{p_\infty} \right) \cos 25^\circ + \left(\frac{p_5}{p_\infty} - \frac{p_2}{p_\infty} \right) \cos 5^\circ \right] \text{ (From Prob. 9.14)}$$

$$c_t = \frac{2}{(1.4)(3)^2} \frac{\ell}{c} [(2.944 + 0.9439) \cos 25^\circ + (1.3888 - 0.6112) \cos 5^\circ]$$

$$c_t = 0.682 \frac{\ell}{c}. \text{ However, } \frac{\ell}{c} = 0.5077 \text{ (From Prob. 9.14)}$$

$$c_t = (0.682)(.5077) = \boxed{0.346}$$

To obtain the

$$c_d = \frac{2}{\gamma M_1^2} \frac{\ell}{c} \left[\left(\frac{p_4}{p_\infty} - \frac{p_3}{p_\infty} \right) \sin 25^\circ + \left(\frac{p_5}{p_\infty} - \frac{p_2}{p_\infty} \right) \sin 5^\circ \right]$$

$$c_d = \frac{2}{(1.4)(3)^2} (.5077) [(2.944 + 0.9439) \sin 25^\circ + (1.3888 - 0.6112) \sin 5^\circ]$$

$$c_d = \boxed{0.1089}$$

Comparison

	Exact (Prob. 9.14)	Linear Theory	% Error
c_t	0.418	0.346	17.2%
c_d	0.169	0.1089	35.6%

To locate point 3

Along the C. Characteristics