11.2 Using the Prandtl - Glauert rule, Calculate the lift coefficient for an NACA 2412 airfoil at $\alpha = 5°$ in a Mach 0.6 freestream.

Known: $\alpha = 5°$, $M = 0.6$

Unknown: c_l

Fig 4.5 @ $\alpha = 5°$

$C_{L,0} \approx 0.75$

$$C_L = \frac{C_{L,0}}{\sqrt{1 - M_\infty^2}}$$

$$\boxed{C_L = 0.9375}$$

11.4 In low-speed incompressible flow, the peak pressure coefficient (at minimum pressure point) on an airfoil is -0.41. Estimate the critical Mach number for this airfoil using the Prandtl-Glauert rule

<u>Known:</u> $C_{p\_min} = -0.41$

Maple analytical solution

$$\boxed{M_{cr} = 0.743}$$

12.1 Consider $\alpha = 5° = 0.0873$ rad.

$$c_\ell = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \frac{4(0.0873)}{\sqrt{(2.6)^2 - 1}} = \boxed{0.1455}$$

From exact theory (Prob. 9.13): $c_\ell = 0.148$

$$\% \text{ error} = \frac{0.148 - 0.1455}{0.148} \times 100 = 1.69\%$$

$$c_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} = c_\ell\, \alpha = (0.1455)(0.0873) = \boxed{0.0127}$$

From exact theory (Prob. 9.13): $c_d = 0.0129$

$$\% \text{ error} = \frac{0.0129 - 0.0127}{0.0129} \times 100 = 1.53\%$$

(b) $\alpha = 15° = 0.2618$ rad

$$c_\ell = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \boxed{0.436}$$

From exact theory (Prob. 9.13): $c_\ell = 0.452$

$$\% \text{ error} = \frac{0.452 - 0.426}{0.452} \times 100 = 3.47\%$$

$$c_d = c_\ell\, \alpha = (0.436)(0.2618) = 0.114$$

From exact theory (Prob. 9.13): $c_d = 0.121$

$$\% \text{ error} = \frac{0.121 - 0.114}{0.121} \times 100 = 5.7\%$$

(c) $\alpha = 30° = 0.5236$ rad

$$c_\ell = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \frac{4(0.5236)}{\sqrt{(2.6)^2 - 1}} = \boxed{0.873}$$

From exact theory (Prob. 9.13): $c_\ell = 1.19$

$$\% \text{ error} = \frac{1.19 - 0.873}{1.19} \times 100 = 26.7\%$$

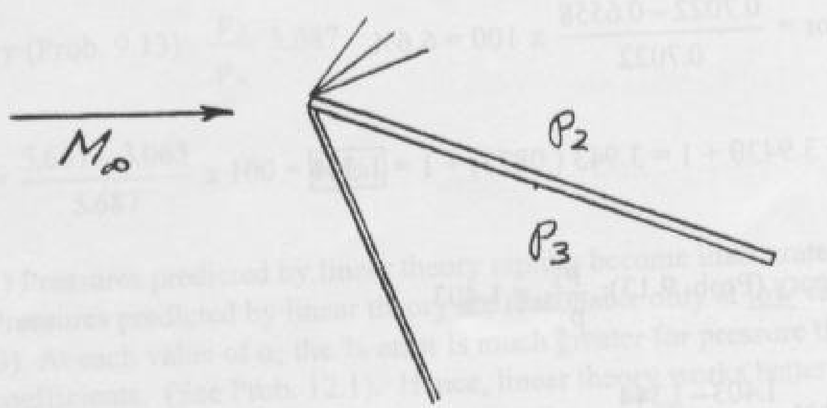
$$c_d = c_\ell\ \alpha = (0.873)(0.5236) = \boxed{0.457}$$

From exact theory (Prob. 9.13): $c_d = 0.687$

$$\% \text{ error} = \frac{0.687 - 0.457}{0.687} = 33.5\%$$

Conclusion: At low α, linear theory is reasonably accurate. However, its accuracy deteriorates rapidly at high α. This is no surprise; we do not expect linear theory to hold for large perturbations. It appears that linear theory is reasonable to at least 5°, and that it is acceptable as high as 15°. At 30° it is unacceptable. Keep in mind that the above comments pertain to the lift and wave drag coefficients only. They say nothing about the accuracy of the pressure distributions themselves.

12.2



Not correct & long

$C_l = \frac{4\alpha}{\tau} = \frac{4(15\pi/180)}{\sqrt{3^2-1}} = .37$

$= 4/\tau\,(\alpha^2 + \Delta\theta_i^2)$

$C_d = \frac{4}{\tau}\left(\left(\frac{15\pi}{180}\right)^2 + \left(\frac{10\pi}{180}\right)^2\right)$

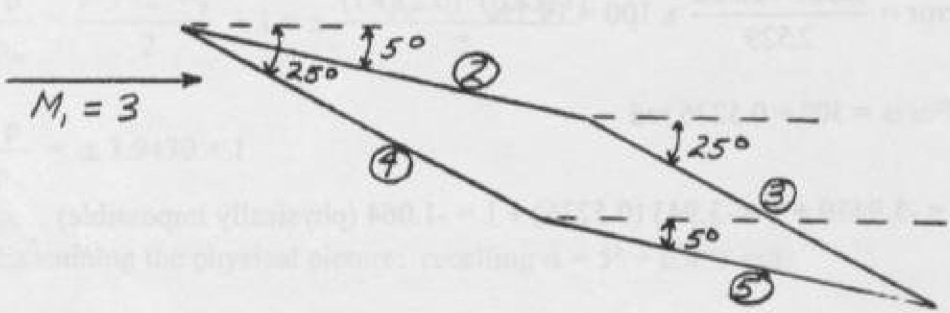
.14

12.3 $\dfrac{p}{p_\infty} = \dfrac{\gamma M_\infty^2 C_p}{2} + 1$ where $C_p = \pm \dfrac{2\theta}{\sqrt{M_\infty^2 - 1}}$

$$C_p = \pm \frac{2\theta}{\sqrt{(3)^2 - 1}} = \pm 0.7071\theta$$

$$\frac{p}{p_\infty} = \pm \frac{(1.4)(3)^2(0.7071)\theta}{2} + 1$$

$$\frac{p}{p_\infty} = \pm 4.455\theta + 1$$



Surface 2: $\theta = 5° = 0.08727$ rad.

$$\frac{p_2}{p_\infty} = -4.455\,(.08727) + 1 = 0.6112$$

Surface 3: $\theta = 25° = 0.4663$ rad

$$\frac{p_3}{p_\infty} = -4.455\,(.4363) + 1 = -0.9439$$

Note: Although a negative pressure is not physically possible, in order to calculate the net force, we must carry it as such.

Surface 4: $\theta = 25° = 0.4363$ rad

$$\frac{p_4}{p_\infty} = 4.455\,(.4363) + 1 = 2.944$$

Surface 5: $\theta = 5° = 0.08727$ rad

$$\frac{p_5}{p_\infty} = 4.455\,(.08727) + 1 = 1.3888$$

$$c_\ell = \frac{2}{\gamma\, M_1^2}\,\frac{\ell}{c}\left[\left(\frac{p_4}{p_\infty} - \frac{p_3}{p_\infty}\right)\cos 25° + \left(\frac{p_5}{p_\infty} - \frac{p_2}{p_\infty}\right)\cos 5°\right] \text{ (From Prob. 9.14)}$$

$$c_\ell = \frac{2}{(1.4)(3)^2}\,\frac{\ell}{c}\,[(2.944 + 0.9439)\cos 25° + (1.3888 - 0.6112)\cos 5°]$$

$$c_\ell = 0.682\,\frac{\ell}{c}.\ \text{However, } \frac{\ell}{c} = 0.5077 \text{ (From Prob. 9.14)}$$

$$c_\ell = (0.682)(.5077) = \boxed{0.346}$$

$$c_d = \frac{2}{\gamma\, M_1^2}\,\frac{\ell}{c}\left[\left(\frac{p_4}{p_\infty} - \frac{p_3}{p_\infty}\right)\sin 25° + \left(\frac{p_5}{p_\infty} - \frac{p_2}{p_\infty}\right)\sin 5°\right]$$

$$c_d = \frac{2}{(1.4)(3)^2}\,(.5077)\,[(2.944 + 0.9439)\sin 25° + (1.3888 - 0.6112)\sin 5°]$$

$$c_d = \boxed{0.1089}$$

Comparison

	Exact (Prob. 9.14)	Linear Theory	% Error
c_ℓ	0.418	0.346	17.2%
c_d	0.169	0.1089	35.6%