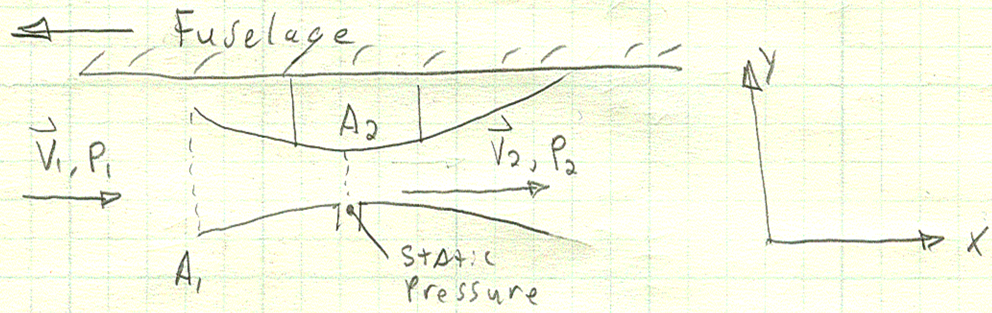
3.2 Consider a venturi with a "throat-to-inlet" area ratio of 0.8, mounted on the side of an airplane fuselage. The airplane is in flight at standard sea level. If the static pressure at the throat is 2100 lb/ft^2, calculate the velocity of the airplane



Known: $P_2 = 2100 \ lb/ft^2$ "Standard sea level"

$\therefore$ 1 atm $= 2116 \ lb/ft^2$

$$\frac{A_2}{A_1} = 0.8 \qquad P_1 = 2116 \ lb/ft^2$$

Unknown: Velocity of airplane V_{air}

Assumptions: $\rho_{air} = 0.002377 \ \frac{slug}{ft^3}$

incomp, $\rho_1 = \rho_2 = $ Const
inviscid

Continuity:

$$\iint_{A_1} \rho_1 \vec{V}_1 \cdot d\vec{S} = \iint_{A_1} \rho_1 (V_1 \cdot n) dA = -\rho_1 V_1 A_1$$

$$\iint_{A_2} \rho_2 \vec{V}_2 \cdot d\vec{S} = \iint_{A_2} \rho_2 (V_2 \cdot n) dA = \rho_2 V_2 A_2$$

$\therefore$ since $\rho_1 = \rho_2 = c$

$$-\rho_1 A_1 V_1 + \rho_2 A_2 V_2 = 0$$

$$A_2 V_2 = A_1 V_1 \qquad V_2 = \frac{A_1}{A_2} V_1$$

<u>Bernoulli's</u>

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$2(P_1 - P_2) = \rho\left[V_2^2 - V_1^2\right]$$

but $V_2 = \frac{A_1}{A_2} V_1$

$$\therefore \quad 2(P_1 - P_2) = \rho\left[\left(\frac{A_1}{A_2}V_1\right)^2 - V_1^2\right]$$

$$2(P_1 - P_2) = \rho\left(\frac{A_1}{A_2}^2 - 1\right)V_1^2$$

$$V_1^2 = \frac{2(P_1 - P_2)}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}$$

$$V_1 = \sqrt{\frac{2(P_1 - P_2)}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}}$$

$$V_1 = \sqrt{\frac{2\left(2116\,\frac{lbf}{ft^2} - 2100\,\frac{lbf}{ft^2}\right)}{\left(0.002377\,\frac{slug}{ft^3}\right)\left[\left(\frac{1}{0.8}\right)^2 - 1\right]}}$$

$$\boxed{V_1 = 154.703\ \frac{ft}{sec}}$$

<u>Note</u>

$$1\ slug = \frac{lbf\text{-}sec^2}{ft}$$

Check units

$$\sqrt{\frac{\frac{lbf}{ft^2}}{\frac{lbf\text{-}sec^2}{ft^4}}}$$

$$\sqrt{\frac{lbf}{ft^2}\,\frac{ft^4}{lbf\ sec^2}}$$

$$\sqrt{\frac{ft^2}{sec^2}} = \frac{ft}{sec}\ \checkmark$$

3.4 Consider a low-speed open-circuit subsonic wind tunnel with an inlet-to-throat area ratio of 12. The tunnel is turned on, and the pressure difference between the inlet (the settling chamber) and the test section is read as a height difference of 10cm on a U-tube mercury manometer. (density of liquid mercury is $1.36 \times 10^4 \frac{kg}{m^3}$)

Calculate the velocity of air in the test section

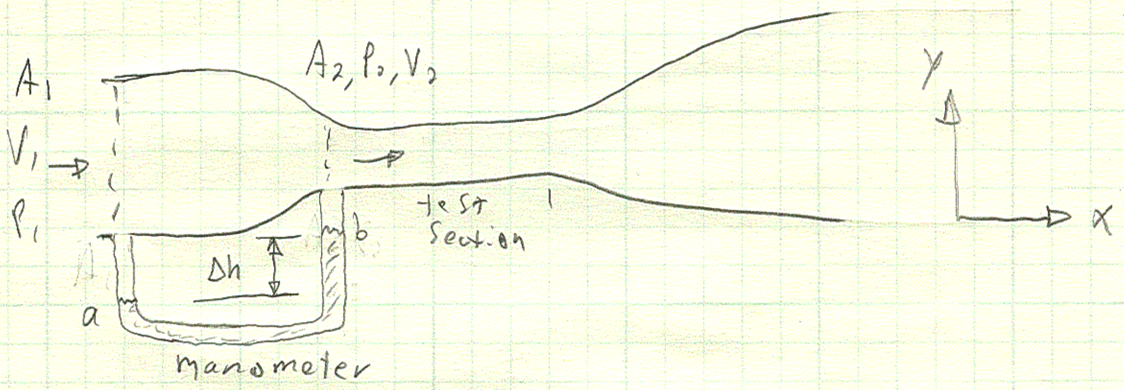
Known: low-speed, open-circuit, subsonic

incomp, inviscid

$A_1/A_2 = 12 \qquad \rho_{mercu} = 1.36 \times 10^4 \ kg/m^3$

$\Delta h_{utube} = 10cm \qquad \rho_{air} = 1.23 \frac{kg}{m^3}$

Unknown: $V_{test\ section}\ (V_2)$



$$P_a + \rho_{mer} g h_a = P_b + \rho_{mer} g h_b$$

$$P_a = P_b + \rho_{mer} g (h_b - h_a) \quad , \quad \Delta h = h_b - h_a$$

Note $P_a = P_1$, $P_b = P_2$

Bernoulli's
$$P_1 + \tfrac{1}{2}\rho V_1^2 = P_2 + \tfrac{1}{2}\rho V_2^2$$

$$2(P_1 - P_2) = \rho_{air}(V_2^2 - V_1^2)$$

but from continuity again $V_1 = \frac{A_2}{A_1} V_2$

$$2(P_1 - P_2) = \rho_{air}\left[V_2^2 - \left(\frac{A_2}{A_1}\right)^2 V_2^2\right]$$

$$2(P_1 - P_2) = \rho_{air}\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] V_2^2$$

$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho_{air}\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]}}$$

but from above

$$P_1 - P_2 = \rho_{merc}\, g\, \Delta h$$

$$V_2 = \sqrt{\frac{2(\rho_{merc}\, g\, \Delta h)}{\rho_{air}\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]}}$$

$$V_2 = \sqrt{\frac{2\left(1.36\times10^4 \frac{kg}{m^3}\right)\left(9.81\ m/sec^2\right)(0.1m)}{\left(1.23\frac{kg}{m^3}\right)\left[1 - \left(\frac{1}{12}\right)^2\right]}}$$

$$\frac{\frac{kg}{m^3}\frac{m}{sec^2}m}{\frac{kg}{m^3}}$$

$$\boxed{V_2 = 147.802\ \frac{m}{s}}$$

$$\frac{\cancel{m^3}\ \cancel{kg}\ m^2}{\cancel{m^3}\ \cancel{kg}\ sec^2}$$

$$\sqrt{\frac{m^2}{sec^2}} = m/s$$

3.6 A pitot tube on an airplane flying at standard sea level reads 1.07×10^5 N/m². What is the velocity of the airplane.

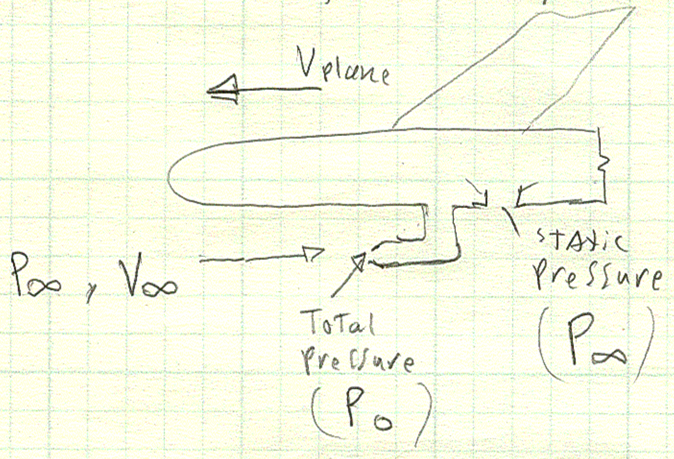
<u>Known:</u> Standard sea level

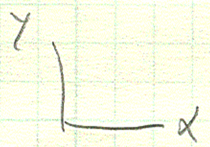
$P = 1\,\text{atm} = 1.01 \times 10^5\ N/m^2$

$\rho = 1.23\ kg/m^3$

$P_{total} = 1.07 \times 10^5\ N/m^2$

<u>Unknown:</u> Velocity of airplane





NO V_0 <u>velocity</u>

$$\underbrace{P_\infty}_{\text{static}} + \underbrace{\tfrac{1}{2}\rho V_\infty^2}_{\text{dynamic}} = \underbrace{P_0}_{\text{total}} + \cancel{\tfrac{1}{2}\rho V_0^2}$$

$$V_\infty^2 = \frac{2(P_0 - P_\infty)}{\rho_{air}}$$

$$V_\infty = \sqrt{\frac{2(P_0 - P_\infty)}{\rho_{air}}} = \sqrt{\frac{2(1.07\times10^5\ N/m^2 - 1.01\times10^5\ N/m^2)}{1.23\ kg/m^3}}$$

$$\boxed{V_\infty = 98.773\ \frac{m}{s}}$$

$$N = \frac{kg \cdot m}{sec^2}$$

$$\sqrt{\frac{\cancel{kg}\, m\, m^3}{\cancel{kg}\, m^2\, sec^2}} = \sqrt{\frac{m^2}{sec^2}} = \frac{m}{s}$$

3.10

- Problem statement

Prove that the velocity potential and the stream function for a uniform flow satisfy Laplace's equation.

- Given. $\phi = V_\infty x \qquad \psi = V_\infty y$
- Unknown: show that ϕ and ψ satisfy Laplace's eq
- Equations: $\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}$, $\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}$

- Solution

$$\left.\begin{array}{l} \frac{\partial\phi}{\partial x} = V_\infty \ ; \ \frac{\partial^2\phi}{\partial x^2} = 0 \\ \frac{\partial\phi}{\partial y} = 0 \ ; \ \frac{\partial^2\phi}{\partial y^2} = 0 \end{array}\right\} \Rightarrow \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$$

$$\left.\begin{array}{l} \frac{\partial\psi}{\partial x} = 0 \ ; \ \frac{\partial^2\psi}{\partial x^2} = 0 \\ \frac{\partial\psi}{\partial y} = V \ ; \ \frac{\partial^2\psi}{\partial y^2} = 0 \end{array}\right\} \Rightarrow \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0$$