MANE 4070: Aerodynamic I

Homework # 5: Solution set

3.15

- Problem statement

Consider a nonlifting flow over a circular cylinder. Derive an expresion for the pressure coefficient at an arbitrary point (r, θ) in this flow and show that it reduces to Eq. (3.101) on the surface of the cylinder.

- Given $\psi = f(r, \theta)$
- Unknown. C_p
- Equations

$$V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \qquad V_\theta = -\frac{\partial \psi}{\partial r}$$

- Solution

$$V_r = V_\infty \cos\theta \left(1 - \frac{R^2}{r^2}\right)$$

$$V_\theta = -V_\infty \sin\theta \left(1 + \frac{R^2}{r^2}\right)$$

$$V^2 = V_r^2 + V_\theta^2 = \left(1 - \frac{R^2}{r^2}\right)^2 V_\infty^2 \cos^2\theta + \left(1 + \frac{R^2}{r^2}\right)^2 V_\infty^2 \sin^2\theta$$

$$C_p = 1 - \frac{V^2}{V_\infty^2} = 1 - \left(1 - \frac{R^2}{r^2}\right)^2 \cos^2\theta - \left(1 + \frac{R^2}{r^2}\right)^2 \sin^2\theta$$

at $r = R$

$$\boxed{C_p = 1 - 4\sin^2\theta}$$

3.16

- Problem statement

Consider the nonlifting flow over a circular cylinder of a given radius, where $V_\infty = 20$ ft/s. If V_∞ is doubled, that is $V_\infty = 40$ ft/s, does the shape of the streamlines change? Explain

- Given ψ, V_r, V_θ
- Unknown: demostrate if streamline shape change by increasing V_∞
- Equations

$$V_r = V_\infty \left(1 - \frac{R^2}{r^2}\right) \cos\theta$$

$$V_\theta = -V_\infty \left(1 + \frac{R^2}{r^2}\right) \sin\theta$$

- Solution.

$$\frac{V_r}{V_\infty} = \left(1 - \frac{R^2}{r^2}\right) \cos\theta \qquad \frac{V_\theta}{V_\infty} = -\left(1 + \frac{R^2}{r^2}\right) \sin\theta$$

At any given point (r, θ), V_r and V_θ are both directly proportional to V_∞. Hence, the direction of the resultant, $\vec{V}$, is the same, no matter what the value of V_∞ may be. Thus, the shape of the streamlines remains the same.

3.17 Consider the lifting flow over a circular cylinder of a given radius and given circulation - if V_∞ is doubled, keeping the circulation the same, does the shape of the streamlines change? Explain.

Knowns: lifting flow, cylinder radius r

Unknown: $\psi = ?$ when $V_\infty = 2V_\infty$

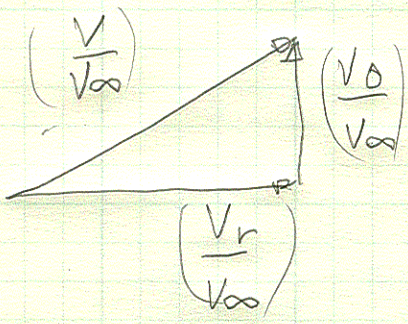
$$V_r = \left(1 - \frac{R^2}{r^2}\right) V_\infty \cos\theta \qquad (3.121)$$

$$V_\theta = -\left(1 - \frac{R^2}{r^2}\right) V_\infty \sin\theta - \frac{\Gamma}{2\pi r} \qquad (3.122)$$

$$\left(\frac{V_r}{V_\infty}\right) = \left(1 - \frac{R^2}{r^2}\right) \cos\theta$$

$$\left(\frac{V_\theta}{V_\infty}\right) = -\left(1 - \frac{R^2}{r^2}\right) \sin\theta - \frac{\Gamma}{2\pi r V_\infty}$$

Since

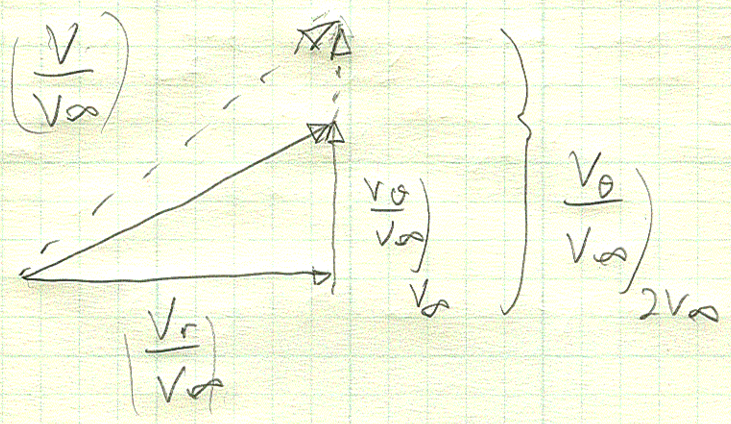


From second term in $\left(\frac{V_\theta}{V_\infty}\right)$

Γ - fixed

$$\frac{-\Gamma}{2\pi r V_\infty} \neq \frac{-\Gamma}{2\pi r\, 2V_\infty} \quad \therefore \quad \left.\frac{V_\theta}{V_\infty}\right|_{V_\infty} \neq \left.\frac{V_\theta}{V_\infty}\right|_{2V_\infty}$$

$\left(\frac{V_r}{V_\infty}\right)$ not changed by $2V_\infty$ but



the resultant velocity vector is changed due to the change in $\left.\frac{V_\theta}{V_\infty}\right)_{2V_\infty}$ therefore

Since the velocity is tangent to the streamline then the streamline must change to satisfy the tangent requirement.