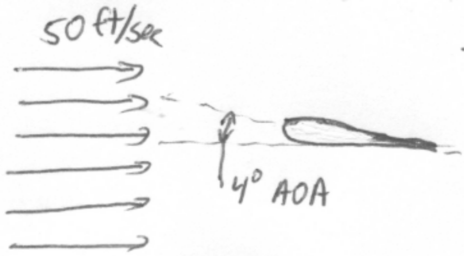
Homework #6 Solutions

4.1) Consider The data for The NACA 2412 in figure 4.5. Calculate The lift and pitching moment about the quarter chord, for This airfoil when The AOA is 4° and The freestream is at standard Sea level conditions with a velocity of 50 ft/sec. The chord is 2 ft.

$$\rho_{SL} = .002377 \frac{slug}{ft^3}$$



from chart 4.5

$C_l \approx .65$

$$L' = \tfrac{1}{2} \rho V_\infty^2 c C_l = \tfrac{1}{2}\left(.002377 \tfrac{slug}{ft^3}\right)\left(50 \tfrac{ft}{sec}\right)^2 (2ft)(.65)$$

$$= 3.86 \frac{lbs}{ft}$$

for The moment

$$M'_{c/4} = \frac{1}{2} \rho V_\infty^2 c^2 C_{m_{c/4}} = \frac{1}{2}\left(.002377 \tfrac{slug}{ft^3}\right)\left(50 \tfrac{ft}{sec}\right)^2 (2ft)^2 (-.04)$$

from chart 4.5

$C_{m_{c/4}} \approx -.04$

$$= -.47 \frac{ft \cdot lbs}{ft}$$

4.4 Starting with Equation (4.35), derive Equation (4.36)

Known:

$$M'_{LE} = -\rho_\infty V_\infty \int_0^c \xi \gamma(\xi)\, d\xi \quad (4.35)$$

Unknown:

$$M'_{LE} = -q_\infty c^2 \frac{\pi \alpha}{2} \quad (4.36)$$

Start with

$$M'_{LE} = -\rho_\infty V_\infty \int_0^c \underset{1}{\xi}\, \underset{2}{\gamma(\xi)}\, \underset{3}{d\xi}$$

recognize need $M'_{LE} = M'_{LE}(\theta)$

From (4.20)

$$\xi = \frac{c}{2}(1 - \cos\theta) \quad (1)$$

From (4.24)

$$\gamma(\theta) = 2\alpha V_\infty \frac{(1+\cos\theta)}{\sin\theta} \quad (2)$$

- Note Limits of integration $0 \to c$, $0 \to \pi$

From (4.22)

$$d\xi = \frac{c}{2}\sin\theta\, d\theta \quad (3)$$

Substitute in

$$M'_{LE} = -\rho_\infty V_\infty \int_0^\pi \frac{c}{2}(1-\cos\theta)\, 2\alpha V_\infty \frac{(1+\cos\theta)}{\sin\theta} \frac{c}{2}\sin\theta\, d\theta$$

$$M'_{LE} = -\rho_\infty V_\infty^2 \frac{c^2}{4} 2\alpha \int_0^\pi (1-\cos\theta)(1+\cos\theta)\, d\theta$$

$$M'_{LE} = -\frac{1}{2}\rho_\infty V_\infty^2 c^2 \alpha \int_0^\pi (1-\cos^2\theta)\, d\theta$$

$$M'_{LE} = -\frac{1}{2}\rho_\infty V_\infty^2 c^2 \alpha \int_0^\pi \sin^2\theta\, d\theta$$

Since $\int \sin^2 x \, dx = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$

$$M'_{LE} = -\frac{1}{2}\rho_\infty V_\infty^2 c^2 \alpha \left[\frac{1}{2}\theta - \frac{1}{4}\sin(2\theta) \Big|_0^\pi \right]$$

$$M'_{LE} = \underbrace{-\frac{1}{2}\rho_\infty V_\infty^2}_{-q_\infty} c^2 \alpha \left[\frac{\pi}{2}\right]$$

$$\boxed{M'_{LE} = -q_\infty c^2 \frac{\pi\alpha}{2}} \quad (4.36)$$

4.5 Consider a thin, symmetric airfoil at 1.5° angle of attack. From the results of thin airfoil theory, calculate the lift coefficient and the moment coefficient about the leading edge.

Known: Symmetric - thin airfoil theory

$\alpha = 1.5°$

Unknown: c_l, and $c_{m,LE}$

From (4.33)

$$c_l = 2\pi\alpha$$

$$c_l = 2\pi \frac{1.5\,\text{deg}}{\left(\frac{180\,\text{deg}}{\pi\,\text{rad}}\right)}$$

$$\boxed{c_l = 0.16449}$$

From (4.39)

$$c_{m,LE} = -\frac{c_l}{4}$$

$$c_{m,LE} = -\frac{0.16449}{4}$$

$$\boxed{c_{m,LE} = -0.0411}$$

MANE 4070: Aerodynamics I
Homework # ~~7~~: solution set

- Problem 4.6

→ Problem statement: The NACA 4412 airfoil has a mean camber line given by:

→ Known:

$$\frac{z}{c}\begin{cases}0.25\left[0.8\frac{x}{c}-\left(\frac{x}{c}\right)^2\right] & \text{for } 0\le\frac{x}{c}\le 0.4\\ 0.111\left[0.2+0.8\frac{x}{c}-\left(\frac{x}{c}\right)^2\right] & \text{for } 0.4\le\frac{x}{c}\le 1\end{cases}$$

Using thin film theory, calculate

→ Unknown: $\alpha_{L=0}$, c_ℓ @ $\alpha=3°$

→ Equations

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi}\frac{dz}{dx}(\cos\theta-1)\,d\theta$$

$$c_\ell = 2\pi(\alpha+\alpha_{L=0})$$

→ Solution.

Determination of $\alpha_{L=0}$

for $0\le\frac{x}{c}\le 0.4$ $\left(\frac{dz}{dx}\right)_1 = 0.2-0.5\left(\frac{x}{c}\right)$

for $0.4\le\frac{x}{c}\le 1$ $\left(\frac{dz}{dx}\right)_2 = 0.0888-0.2222\left(\frac{x}{c}\right)$

Since $x=\frac{c}{2}(1-\cos\theta)$, then:

$$\left(\frac{dz}{dx}\right)_1 = -0.05+0.25\cos\theta,\quad \text{for } 0\le\theta\le 1.3694$$

$$\left(\frac{dz}{dx}\right)_2 = -0.0223+0.1111\cos\theta,\quad \text{for } 1.3694\le\theta\le\pi$$

(2)

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}(\cos\theta - 1)\,d\theta = \frac{1}{\pi}\int_0^{1.3694} (-0.05 + 0.25\cos\theta)(\cos\theta - 1)\,d\theta$$

$$-\frac{1}{\pi}\int_{1.3694}^{\pi} (-0.0223 + 0.1111\cos\theta)(\cos\theta - 1)\,d\theta = -\frac{1}{\pi}\int_0^{1.3694} (0.05 - 0.3\cos\theta$$

$$+0.25\cos^2\theta)\,d\theta - \frac{1}{\pi}\int_{1.3694}^{\pi} (0.0223 - 0.13334\cos\theta + 0.1111\cos^2\theta)\,d\theta$$

$$= -\frac{1}{\pi}\left[0.05\,\theta - 0.3\sin\theta + 0.25\left(\frac{\theta}{2} + \frac{1}{4}\sin 2\theta\right)\right]_0^{1.3694} - \frac{1}{\pi}\Big[$$

$$0.0223\theta - 0.1334\sin\theta + 0.111\left(\frac{\theta}{2} + \frac{1}{4}\sin 2\theta\right)\Big]_{1.3694}^{\pi} =$$

$$= -\frac{1}{\pi}\left[0.06847 - 0.2939 + 0.1712 + 0.0245\right] - \frac{1}{\pi}\Big[0.0701$$

$$+0.1745\Big] + \frac{1}{\pi}\left[0.0305 - 0.1307 + 0.0761 + 0.0109\right] =$$

$$= \frac{-0.2281}{\pi} = -0.0726\text{ rad} = \boxed{-4.16^\circ}$$

Determination of c_ℓ

$$c_\ell = 2\pi(\alpha + \alpha_{L=0}) = \frac{2\pi}{57.3}\left[3 - (-4.16)\right] = \boxed{0.782}$$

Problem 4.7

→ Problem statement: for the airfoil given in Problem 4.6, calculate $C_{m,c/4}$ and x_{cp}/c when $\alpha = 3^\circ$

→ Known: z/c given in 4.6
$\alpha = 3^\circ$

→ Unknown: $C_{m,c/4}$ and x_{cp}/c

→ Equations

$$A_{1,2} = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx}\cos\theta\, d\theta \qquad C_{m_{c/4}} = \frac{\pi}{4}(A_2 - A_1)$$

$$\frac{x_{cp}}{c} = \frac{1}{4}\left[1 + \frac{\pi}{c_l}(A_1 - A_2)\right]$$

→ Solution

$$A_1 = \frac{2}{\pi}\int_0^{1.3694}(0.05 + 0.25\cos\theta)\cos\theta\, d\theta + \frac{2}{\pi}\int_{1.3694}^{\pi}(-0.0223 + 0.1111\cos\theta)\cos\theta\, d\theta = \frac{2}{\pi}\int_0^{1.3694}(-0.05\cos\theta + 0.25\cos^2\theta)\, d\theta + \frac{2}{\pi}\int_{1.3694}^{\pi}(-0.0223\cos\theta + 0.1111\cos^2\theta)\, d\theta = \frac{2}{\pi}\left[-0.05\sin\theta + 0.25\left(\frac{\theta}{2} + \frac{1}{4}\sin 2\theta\right)\right]_0^{1.3694} + \frac{2}{\pi}\left[(-0.0223)\sin\theta + 0.1111\left(\frac{\theta}{2} + \frac{1}{4}\sin 2\theta\right)\right]_{1.3694}^{\pi} = \frac{2}{\pi}\left[-0.04899 + 0.25(0.6847 + 0.098) + 0.1745\right.$$

$$+0.02185-0.1111(0.6847+0.098)\Big)\Big]$$

$$A_1 = 0.2561\ \frac{2}{\pi} = 0.163$$

$$A_2 = \frac{2}{\pi}\int_0^{1.3694}(-0.05+0.25\cos\theta)\cos 2\theta\, d\theta + \frac{2}{\pi}\int_{1.3694}^{\pi}(-0.0223+0.1111$$

$$\cos\theta)\cos\theta\, d\theta = \frac{2}{\pi}\Big[\frac{1}{2}(-0.05)\sin 2\theta + 0.25\Big(\frac{\sin\theta}{2}+\frac{\sin 3\theta}{6}\Big)\Big]_0^{1.36}$$

$$+\frac{2}{\pi}\Big[\frac{1}{2}(-0.0223)\sin 2\theta + 0.1111\Big(\frac{\sin\theta}{2}+\frac{\sin 3\theta}{6}\Big)\Big]_{1.3694}^{\pi}$$

$$= \frac{2}{\pi}\Big[-0.0098+0.25(0.4899-0.1372)+0.004371-0.1111(0.4899$$

$$-0.1372)\Big] = 0.0436\ \frac{2}{\pi} - 0.0277$$

$$C_{m_{c/4}} = \frac{\pi}{4}(A_2 - A_1) = \frac{\pi}{4}(0.0277-0.163) = \boxed{-0.1063}$$

$$\frac{x_{cp}}{c} = \frac{1}{4}\Big[1+\underbrace{\frac{\pi}{}}_{0.782}(0.163-0.0277)\Big] = \boxed{0.386}$$