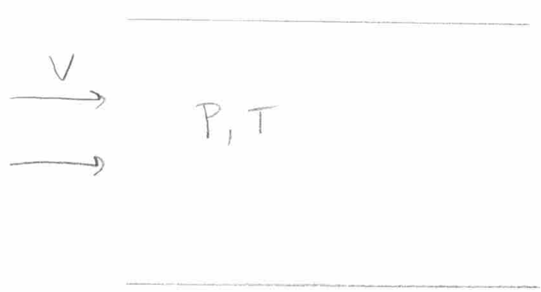
8.3 At a given point in the flow, T = 300 K, P = 1.2 atm, and V = 250 M/s. At this point, calculate the corresponding values of P_0, T_0, P^*, T^*, and M^*.

Known: T = 300 K, P = 1.2 atm, V = 250 M/s, $\gamma = 1.4$, $R = 0.287 \frac{kJ}{kg\,K}$

unknown: P_0, T_0, P^*, T^* and M^*



Assume isentropic

Eqns:

$$a = \sqrt{\gamma R T}$$

$$M = \frac{u}{a}$$

$$\frac{P_0}{P} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\gamma/\gamma-1}$$

$$\frac{T^*}{T_0} = \frac{2}{\gamma+1}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2$$

$$M^{*2} = \frac{(\gamma+1) M^2}{2 + (\gamma-1) M^2}$$

$$\frac{P^*}{P_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

Solutions:

$$a = \sqrt{(1.4)(287 \tfrac{J}{kg\,K})(300K)} = 347.188 \text{ M/s}$$

$$M = \frac{250 \text{ M/s}}{347.188 \text{ M/s}} = 0.72$$

$$P_0 = 1.2 \text{ atm}\left[1 + \frac{0.4}{2}(0.72)^2\right]^{\frac{1.4}{0.4}} = \boxed{1.6949 \text{ atm}}$$

$$T_0 = 300 K\left[1 + \frac{0.4}{2}(0.72)^2\right] = \boxed{331.1 \text{ K}}$$

(8.3 cont)

$$P^* = 1.6949 \text{ atm} \left(\frac{2}{2.4}\right)^{\frac{1.4}{0.4}} = \boxed{0.8954 \text{ atm}}$$

$$T^* = 331.1 \text{ K} \left(\frac{2}{2.4}\right) = \boxed{275.92 \text{ K}}$$

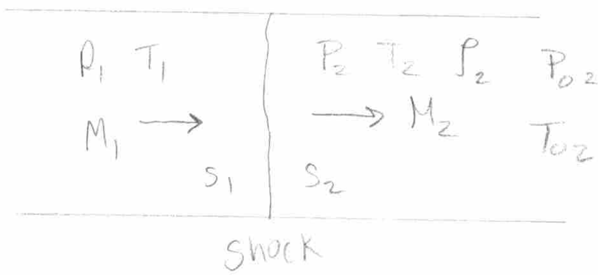
$$M^* = \sqrt{\frac{2.4\ (0.72)^2}{2 + 0.4(0.72)^2}} = \boxed{0.7508}$$

8.7. The flow just upstream of a normal shockwave is given by $P_1 = 1\text{atm}$, $T_1 = 288\text{K}$, $M_1 = 2.6$. Calculate the following properties just downstream of the shock: P_2, T_2, ρ_2, M_2, $P_{0,2}$, T_{02} and Δs entropy across the shock.

Known: $P_1 = 1\text{atm}$ $\quad T_1 = 288\text{K}$ $\quad M_1 = 2.6$ $\quad \gamma = 1.4$

$R = 287 \ \text{J/kg K}$

unknown: P_2, T_2, ρ_2, M_2, $P_{0,2}$, T_{02}, Δs across shock



Eqns

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)$$

$$\rho_1 = \frac{P_1}{RT_1}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)\left(\frac{\rho_1}{\rho_2}\right)$$

$$M_2^2 = \frac{1 + \left(\frac{\gamma-1}{2}\right)M_1^2}{\gamma M_1^2 - \left(\frac{\gamma-1}{2}\right)}$$

$$\frac{P_{0,2}}{P_2} = \left(1 + \left(\frac{\gamma-1}{2}\right)M_2^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{T_{02}}{T_2} = 1 + \frac{\gamma-1}{2}M_2^2$$

$$\frac{P_{01}}{P_1} = \left(1 + \left(\frac{\gamma-1}{2}\right)M_1^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{02}}{P_{01}} = e^{-\frac{(s_2 - s_1)}{R}}$$

Solutions

$$\frac{P_2}{P_1} = \left[1 + \frac{2(1.4)}{2.4}(2.6^2 - 1)\right] = 7.72$$

$$\boxed{P_2 = 7.72\,\text{atm}}$$

$$\rho_1 = \frac{1\text{atm}\left(\frac{101.325\text{ kPa}}{\text{atm}}\right)}{\left(0.287\frac{\text{kJ}}{\text{kg K}}\right)(288\text{K})} = 1.226\ \frac{\text{kg}}{\text{m}^3}$$

(8.7 cont.)

$$\frac{\rho_2}{\rho_1} = \frac{2.4(2.6)^2}{2+0.4(2.6)^2} = 3.45 \qquad \rho_2 = \left(1.226\ \tfrac{kg}{m^3}\right)(3.45) = \boxed{4.23\ \tfrac{kg}{m^3}}$$

$$T_2 = (288\ K)(7.72)\left(\frac{1}{3.45}\right) = \boxed{644.45\ K}$$

$$M_2 = \sqrt{\frac{1+\frac{0.4}{2}(2.6)^2}{1.4(2.6)^2-\frac{0.4}{2}}} = \boxed{0.504}$$

$$P_{0,2} = (7.72\ atm)\left(1+\frac{0.4}{2}(0.504)^2\right)^{\frac{1.4}{0.4}} = \boxed{9.182\ atm}$$

$$T_{0,2} = (644.45\ K)\left(1+\frac{0.4}{2}(0.504)^2\right) = \boxed{677.19\ K}$$

$$P_{01} = (1\ atm)\left(1+\frac{0.4}{2}(2.6)^2\right)^{\frac{1.4}{0.4}} = 19.954\ atm$$

$$\ln\frac{P_2}{P_1} = -\frac{S_2-S_1}{R}$$

$$S_2-S_1 = -R\ln\frac{P_2}{P_1} = -\left(287\ \tfrac{J}{kg\,K}\right)\ln\left(\frac{9.182\ atm}{19.954\ atm}\right) = \boxed{222.76\ \tfrac{J}{kg\,K}}$$

8.14 Derive the Rayleigh Pitot tube formula, Eqn (8.80)

Known:

$$\frac{P_{02}}{P_2} = \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$M_2^2 = \frac{1 + \left(\frac{\gamma-1}{2}\right) M_1^2}{\gamma M_1^2 - \left(\frac{\gamma-1}{2}\right)}$$

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)$$

Unknown: $\frac{P_{02}}{P_1}$ Eqn (8.80)

Solution:

$$\frac{P_{02}}{P_1} = \frac{P_{02}}{P_2}\frac{P_2}{P_1}$$ use formulas for each ratio

$$\frac{P_{02}}{P_1} = \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}}\left(1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)\right)$$ put in terms of M_1

$$\frac{P_{02}}{P_1} = \left(1 + \frac{\gamma-1}{2}\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{\gamma M_1^2 - \left(\frac{\gamma-1}{2}\right)}\right)\right)^{\frac{\gamma}{\gamma-1}}\left(1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)\right)$$ for each term, simplify & find common denominator

$$\frac{P_{02}}{P_1} = \left(\frac{2\gamma M_1^2 - (\gamma-1)}{2\gamma M_1^2 - (\gamma-1)} + \frac{\gamma - 1 + \frac{1}{2}(\gamma-1)^2 M_1^2}{2\gamma M_1^2 - (\gamma-1)}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{\gamma+1}{\gamma+1} + \frac{2\gamma M_1^2 - 2\gamma}{\gamma+1}\right)$$ add terms and multiply out $(\gamma-1)^2$

$$\frac{P_{02}}{P_1} = \left(\frac{2\gamma M_1^2 - (\gamma-1) + \gamma - 1 + \frac{1}{2}M_1^2(\gamma^2 - 2\gamma + 1)}{2\gamma M_1^2 - (\gamma-1)}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{1 - \gamma + 2\gamma M_1^2}{\gamma+1}\right)$$ multiply by $\left(\frac{2}{2}\right)^{\frac{\gamma}{\gamma-1}}$ and simplify

$$\frac{P_{02}}{P_1} = \left(\frac{M_1^2(\gamma^2 + 2\gamma + 1)}{4\gamma M_1^2 - 2(\gamma-1)}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{1 - \gamma + 2\gamma M_1^2}{\gamma+1}\right)$$ simplify quadratic

$$\frac{P_{02}}{P_1} = \left(\frac{M_1^2(\gamma+1)^2}{4\gamma M_1^2 - 2(\gamma-1)}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{1 - \gamma + 2\gamma M_1^2}{\gamma+1}\right) = \text{Eqn (8.80)}$$