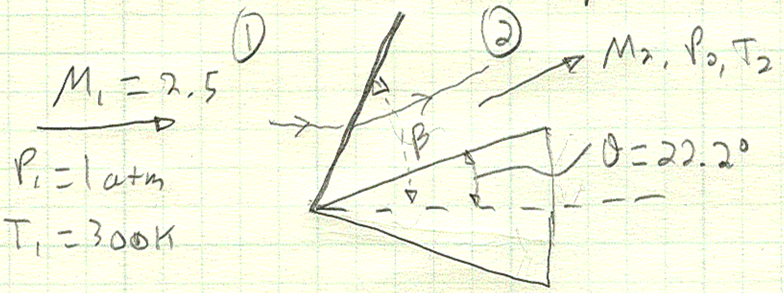
9.5 Consider the flow over a 22.2° half angle wedge. If $M_1 = 2.5$, $P_1 = 1$ atm and $T_1 = 300K$ Calculate the wave angle and P_2, T_2, M_2

Known: $M_1 = 2.5$, $P_1 = 1$ atm $T_1 = 300K$

22.2° half-angle wedge

Unknown: Wave angle β, P_2, T_2, M_2



With $M_1 = 2.5$ $P_1 = 1atm$ $T_1 = 300K$

$$\tan(\theta) = 2\cot(\beta)\,\frac{M_1^2 \sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}$$

$$\tan(22.2) = 2\cot(\beta)\,\frac{(2.5)^2 \sin^2\beta - 1}{(2.5)^2[1.4 + \cos(2\beta)] + 2}$$

Maple eqbtm

$$\beta = 0.80 \text{ rad} \equiv 45.887 \text{ deg}$$

$$M_{n_1} = M_1 \sin(\beta) = 2.5 \sin(22.2)$$

Maple - $M_{n_1} = 1.794$

$$M_{n_2} = \left(\frac{1 + \frac{(\gamma - 1)}{2} M_{n_1}^2}{\gamma M_{n_1}^2 - \frac{1}{2}\gamma + \frac{1}{2}}\right)^{1/2}$$

maple $M_{n2} = 0.6176$

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma(M_{n1}^2 - 1)}{\gamma + 1}$$

maple $\frac{P_2}{P_1} = 3.592$

$$P_2 = \left(\frac{P_2}{P_1}\right) P_1 = 1\,atm\,(3.592)$$

$$\boxed{P_2 = 3.592\ atm}$$

$$\frac{T_2}{T_1} = \frac{\left[1 + \frac{2\gamma(M_{n1}^2 - 1)}{\gamma+1}\right]\left(2 + (\gamma-1) M_{n1}^2\right)}{(\gamma+1) M_{n1}^2}$$

maple $\frac{T_2}{T_1} = 1.52779$

$$T_2 = \frac{T_2}{T_1} T_1 = (300\,K)(1.52779)$$

$$\boxed{T_2 = 458.337\,K}$$

$$M_2 = \frac{M_{n2}}{\sin(\beta - \theta)} = \frac{0.6176}{\sin(45.88 - 22.2)}$$

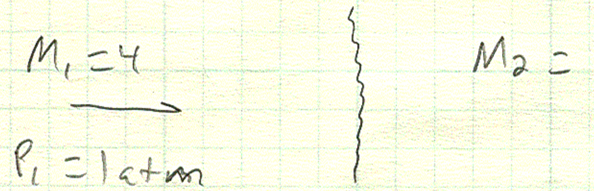
Maple $\boxed{M_2 = 1.537}$

9.8 Consider a Mach 4 airflow at a pressure of 1atm. We wish to slow this flow to subsonic speed through a system of shock waves with as small a loss in "Total pressure" as possible. Compare the loss in total pressure for the following three shock systems

a) normal shock
b) oblique with deflection of 25.3°, then normal shock
c) oblique @ 25.3°, oblique @ 20°, normal shock
- induce about the efficiency of these systems

a)

<u>Known</u>: $M_1 = 4$ $\quad P_1 = 1\,atm$



<u>unknown</u>: $P_{0,1}$, $P_{0,2}$ $\quad M_2$

For normal shock

$$M_2 = \left(\frac{1 + \frac{(\gamma-1)M_1^2}{2}}{\gamma M_1^2 - \frac{1}{2}\gamma + \frac{1}{2}} \right)^{1/2}$$

Maple $\quad M_2 = 0.435$

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma(M_1^2 - 1)}{\gamma + 1}$$

Maple $\quad \frac{P_2}{P_1} = 18.50$

$$P2 = P_1 \frac{P_2}{P_1} = 1atm\,(18.5) \qquad P_2 = 18.5\,atm$$

isentropic relations at state 2

$$M_2 = 0.435$$

$$\frac{P_{0,2}}{P_2} = \left(1 + \frac{(\gamma-1)M_2^2}{2}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}$$

Maple

$$\frac{P_{0,2}}{P_2} = 1.138$$

$$P_{0,2} = P_2\left(\frac{P_{0,2}}{P_2}\right) = (18.5\,atm)(1.138)$$

$$P_{0,2} = 21.06\ atm$$

Maple

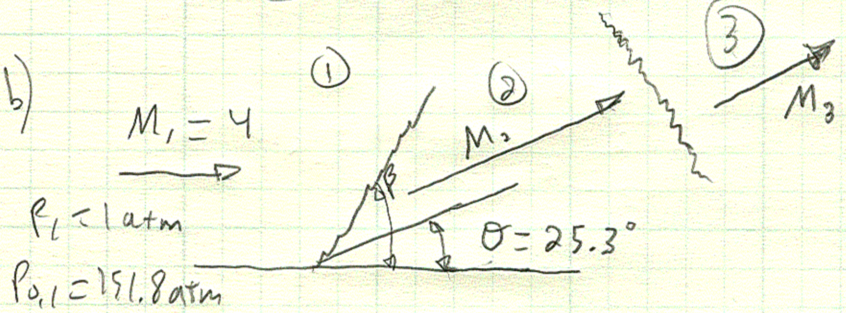
$$P_{0,1} = 151.83\ atm$$

$$\therefore\quad \Delta P_0 = P_{0,1} - P_{0,2}$$

$$= (151.83 - 21.06)\,atm$$

$$\boxed{\Delta P_0 = 130.77\ atm}$$

b)



With $M_1 = 4$, $\theta = 25.3°$

Maple using β-θ-M

$$\beta_1 = 38.84\ deg$$

$$M_{n_1} = M_1 \sin(\beta_1)$$

$$M_{n_1} = 2.51$$

now using $\underline{M_{2n}\_NS}$ with $M1$ as "M_{n_1}" for "Oblique shock - not normal shock"

$$M_{2n} = \left(\frac{1 + \frac{(\gamma-1)}{2} (M_{n1})^2}{\gamma \cdot M_{n_1}^2 - \frac{1}{2}\gamma + \frac{1}{2}} \right)^{1/2}$$

Maple

$$M_{n2} = 0.512$$

$$M_2 = \frac{M_{n2}}{\sin(\beta_1 - \theta)}$$

$$\underline{M_2 = 2.188}$$

Apply isentropic relations

Maple $\frac{P_2}{P_1} = 7.175$

$$\frac{P_{0,2}}{P_2} = 10.487$$

$$\frac{P_{0,2}}{P_{0,1}} = \frac{P_{0,2}}{P_2} \left(\frac{P_2}{P_1}(M_{n1})\right) \frac{P_1}{P_{0,1}} = 0.4956$$

Next 2 → 3 across normal shock

$$M_3 = \left(\frac{1 + \frac{(\gamma-1)}{2} M_2^2}{\gamma M_2^2 - \frac{1}{2}\gamma + \frac{1}{2}} \right)^{1/2}$$

Maple $\underline{M_3 = 0.5487}$

Apply isentropic relations

$$\frac{P_3}{P_2} = 5.416$$

$$\frac{P_{0,3}}{P_3} = 1.227$$

$$\frac{P_{0,3}}{P_{0,2}} = 0.6337$$

$\therefore$ Maple $\quad P_{0,3} = 47.69$ atm

$$\Delta P_0 = P_{0,1} - P_{0,3} = 151.8 - 47.69$$

$$\boxed{\Delta P_0 = 104.144 \text{ atm}}$$

c) Same process but adding 2nd oblique shock

Maple $\quad P_{0,4} = 63.79$ atm $\qquad M_4 = 0.742$

$$\Delta P_0 = P_{0,1} - P_{0,4} = (151.8 - 63.79) \text{ atm}$$

$$\boxed{\Delta P_0 = 88.044 \text{ atm}}$$

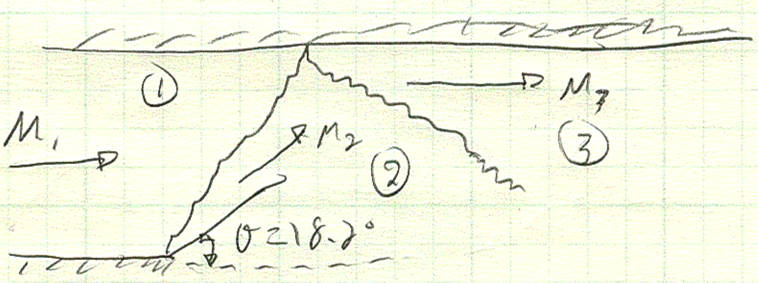
the most efficient method to minimize total pressure loss is the combination of several oblique shocks and a normal shock system <u>C</u>

9.9 Consider an oblique shock generated at a compression corner with a deflection angle $\theta = 18.2°$. A straight horizontal wall is present above the corner, as shown in Fig 9.17. If the upstream flow has the properties $M_1 = 3.2$, $p_1 = 1\text{atm}$ and $T_1 = 520°R$ Calculate M_3, p_3 and T_3 behind the reflected shock from the upper wall. Also, obtain the angle Φ which the reflected shock makes with the upper wall.

Known: $M_1 = 3.2$ $P_1 = 1\text{atm}$ $T_1 = 520°R$

Unknown: M_3, P_3, T_3, Φ

Fig 9.17



from ① to ② oblique shock

Maple βθM $\beta_1 = 34.29°$

$M_2 = 2.22$

isentropic relations

$$\frac{P_2}{P_1} = 3.63 \qquad \frac{T_2}{T_1} = 1.534$$

next ② to ③ across "reflected" oblique shock

$M_2 = 2.22$ $\theta_2 = 18.2°$

$\beta_2 = 44.95°$

Maple

$$\boxed{M3 = 1.51}$$

From geometry

$$\Phi = \beta_2 - \theta_2$$

$$\Phi = 44.95 - 18.2$$

$$\boxed{\Phi = 26.75^\circ}$$

Isentropic relations

$$\frac{P_3}{P_2} = 2.71$$

$$P_3 = \left(\frac{P_3}{P_2}\right)\left(\frac{P_2}{P_1}\right)P_1 \qquad \boxed{P_3 = 9.816 \text{ atm}}$$

$$T_3 = \left(\frac{T_3}{T_2}\right)\left(\frac{T_2}{T_1}\right)T_1 \qquad \boxed{T_3 = 1090.36\ ^\circ R}$$

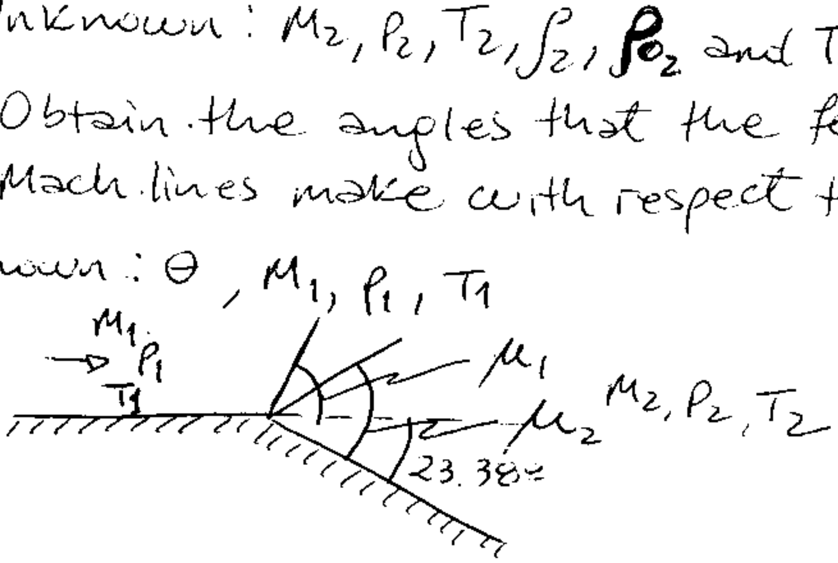
(9.10)

- Problem statement: consider the supersonic flow over an expansion corner, such as given in Fig 9.23. The deflection angle is 23.38°. If the flow upstream of the corner is given by $M_1 = 2$; $P_1 = 0.7$ atm, $T_1 = 630R$ Calculate,

- Unknown: $M_2, P_2, T_2, \rho_2, P_{o_2}$ and T_{o_2}

  Obtain the angles that the forward and rearward Mach lines make with respect to the upstream direction

- Known: θ, M_1, P_1, T_1



- Solution

From table A.3, for $M_1 = 2$, $\nu_1 = 26.38°$

$$\nu_2 = \theta + \nu_1 = 49.76^\circ$$

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \arctan\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \arctan\left(\sqrt{M^2-1}\right)$$

$$\therefore \nu_2 = \nu(M_2) = 49.76°$$

Solving the above eq. gives $\boxed{M_2 = 3}$

from table A.1,

for $M_1 = 2 \rightarrow \frac{P_{o1}}{P_1} = 7.824$, $\frac{T_{o1}}{T_1} = 1.8$

$M_2 = 3 \rightarrow \frac{P_{o2}}{P_2} = 36.73$, $\frac{T_{o2}}{T_2} = 2.8$

Isentropic flow $\Rightarrow$ $p_{01} = p_{02}$ and $T_{01} = T_{02}$

$$p_2 = \frac{p_2}{p_{02}} \cdot \frac{p_{01}}{p_1} \cdot p_1 = \left(\frac{1}{36.73}\right)(7.824)(0.7) = \boxed{0.149 \text{ atm}}$$

$$T_2 = \frac{T_2}{T_{02}} \frac{T_{01}}{T_1} \cdot T_1 = \left(\frac{1}{2.8}\right)(1.8)(630) = \boxed{405°R}$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{(0.149)(2116)}{(1716)(405)} = \boxed{4.537 \times 10^{-4} \text{ slug/ft}^3}$$

$$p_{02} = p_{01} = \frac{p_{01}}{p_1} p_1 = (7.824)(0.7) = \boxed{5.477 \text{ atm}}$$

$$T_{02} = T_{01} = \frac{T_{01}}{T_1} \cdot T_1 = (1.8)(630) = \boxed{1134 R}$$

From table A.3, for $M_1 = 2$ $\mu_1 = 30°$

for $M_2 = 3$ $\mu_2 = 19.47°$

Referenced to the upstream direction,

Angle of forward Mach line $= \mu_1 = \boxed{30°}$

Angle of rearward Mach line $= \mu_2 - \theta = \boxed{-3.91°}$

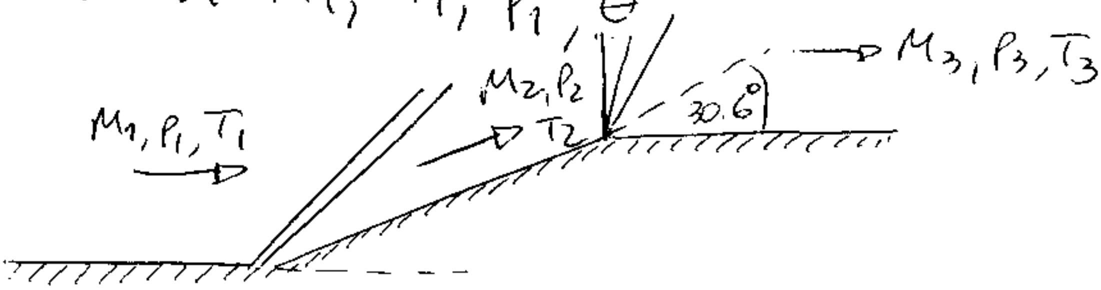
Note: the rearward Mach line is below the upstream direction

(9.12) (8)

- Problem statement: A supersonic flow at $M_1 = 3$, $T_1 = 285$ and $P_1 = 1$ atm is deflected upward through a compression corner with $\theta = 30.6^\circ$ and then is subsequently expanded around a corner of the same angle such that the flow direction is the same as its original direction.

- Unknown: M_3, P_3, T_3.
- Since the resulting flow is in the same direction as the original flow, would you expect $M_3 = M_1$, $P_3 = P_1$ and $T_3 = T_1$? Explain.

- Known M_1, T_1, P_1, θ



From θ-β-M diagram.

for $M_1 = 3$ and $\theta = 30.6^\circ$ $\beta = 53.1^\circ$

$$M_{n_1} = M_1 \sin\beta = 3 \sin 53.1 = 2.4$$

Table A.2, for $M_{n_1} = 2.4 \rightarrow \frac{P_2}{P_1} = 6.553,\ \frac{T_2}{T_1} = 2.04,\ \frac{P_{02}}{P_{01}} = 0.5$

$$M_{n_2} = 0.531$$

$$M_2 = \frac{M_{n_2}}{\sin(\beta - \theta)} = \frac{0.5231}{\sin(53.1 - 30.6)} = 1.37$$

From table A.3, for $M_2 = 1.37$, $\nu_2 = 8.128$

$$\nu_3 = 8.128 + 30.6 = 38.73^\circ$$

From A.3 → for $\nu_3 = 38.73^\circ$, $M_3 = 2.48$

From A.1 → for $M_1 = 3$, $\frac{P_{01}}{P_1} = 36.73$, $\frac{T_{01}}{T_1} = 2.8$

for $M_3 = 2.48$, $\frac{P_{03}}{P_3} = 15.156$, $\frac{T_{03}}{T_3} = 2.23$

$$P_3 = \frac{P_3}{P_{03}} \cdot \frac{P_{03}}{P_{02}} \frac{P_{02}}{P_{01}} \frac{P_{01}}{P_1} P_1 = \left(\frac{1}{16.56}\right)(1)(0.5401)(36.73)(1\,atm)$$

$$= \boxed{120\ atm}$$

$$T_3 = \frac{T_3}{T_{03}} \frac{T_{03}}{T_{02}} \frac{T_{02}}{T_{01}} \frac{T_{01}}{T_1} T_1 = \left(\frac{1}{2.23}\right)(1)(1)(2.8)(285)$$

$$= \boxed{357.8\ K}$$

We can observe a difference between parameters of zone 1 and 3 because there is an entropy increase across the shock wave.