Application of Crystal Plasticity to Fatigue Damage Modeling for Al 7075-T651

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Sponsor: DARPA SIPS program (Northrop-Grumman)
Outline

★ Motivation
★ Methodology
  ★ Constitutive model
  ★ Finite element formulation
  ★ Metrics for damage accumulation
★ Implementation
★ Results
  ★ Calibration results
  ★ Example: 45-grain polycrystal
★ Future work
Motivation

★ Fatigue cracks are observed to initiate primarily at cracked particles in Al 7075-T651
★ The length of fatigue cracks is not observed to be a function of particle size
★ Approach ⇒ focus on microstructure

[Wei and Harlow]
Multiscale Nature of Problem

[Wei and Harlow, Weiland, Wiezorek]
Collaborative Approach

CMU, MSU, NG
Lehigh, Alcoa

Experimental
Observations

Underlying
Phenomena

RPI

Crystal
Constitutive
Model

Orowan Looping

Cornell, RPI

Polycrystal
Model

WCCM 2006 – slide 5/19
Methodology

Constitutive Model

Deformation gradient

\[ F = e^F F^pF \]

Green strain tensor

\[ eE = \frac{1}{2} (e F^T e F - I) \]

Hyperelastic potential

\[ \psi = \frac{1}{2} eE : C : eE \]

Second Piola-Kirchoff stress

\[ S = C : eE \]

Anisotropic elasticity

\[ C_{ijkn} = C_{jikn} = C_{ijnk} = C_{knij} \]

[Kocks (1966), Hart (1972), Schmitt, Lipinski, Berveiller (1997), Han, Wagoner and Barlat (2004)]
Methodology

**Slip Model**

Plastic velocity gradient

\[ p \hat{L} = \sum_{\alpha=1}^{12} \dot{\gamma}^\alpha P^\alpha \]

Schmid tensor

\[ P^\alpha = s^\alpha \otimes m^\alpha \]

Slip rate

\[ \dot{\gamma}^\alpha = \dot{\gamma}_o \frac{\tau^\alpha}{g^\alpha} \left| \frac{\tau^\alpha}{g^\alpha} \right|^{\frac{1}{m}-1} \]

Hardness evolution

\[ \dot{g}^\alpha = G_o \left( \frac{g_o - g^\alpha}{g_s - g_o} \right) H^{\alpha\beta} \]

Projection factor

\[ H^{\alpha\beta} = \sum_{\beta} 2 \left| S^\alpha_{ij} S^\beta_{ij} \left| \dot{\gamma}^\beta \right| \right| \]

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Finite Element Formulation

Governing equations:

\[
\left(\sigma'_{ij} + p \delta_{ij}\right)_{,j} = 0, \quad \frac{1}{3}\sigma_{ii} - p = 0
\]

Weak forms (total Lagrangian):

\[
\int_{\Omega_o} \left(\sigma'_{ij} + p^h \delta_{ij}\right) \psi_{\alpha,K} F_{K}^{-1} J d\Omega_o - \int_{\partial\Omega_{2o}} \hat{t}_i \psi_{\alpha} \frac{dA}{dA_o} d\Gamma_o = 0
\]

\[
\int_{\Omega_o} \frac{1}{K} \left(\frac{1}{3}\sigma^h_{ii} - p^h\right) \tilde{\psi}_r J d\Omega_o = 0
\]

\[
f^\text{int}_{i\alpha}(\bar{\mathbf{u}},\bar{\mathbf{p}})
\]

\[
f^\text{ext}_{i\alpha}(\bar{\mathbf{u}},\bar{\mathbf{p}})
\]
Methodology

Finite Element Formulation

Linearized equations:

\[
K^r_{i\alpha j\beta} \Delta \bar{u}_{j\beta} + G^r_{i\alpha \varphi} \Delta \bar{p}_\varphi = f^\text{ext}_{i\alpha} - f^\text{int}_{i\alpha} (\bar{u}^r, \bar{p}^r)
\]
\[
H^r_{\rho j\beta} \Delta \bar{u}_{j\beta} + M^r_{\rho \varphi} \Delta \bar{p}_\varphi = 0 - h_\rho (\bar{u}^r, \bar{p}^r)
\]

Discontinuous pressure field allows for a \( \Delta \bar{p} \) solution on the element level:

\[
\Delta \bar{p}_\varphi = -M^{-1}_{\varphi \rho} (h_\rho (\bar{u}^r, \bar{p}^r) + H^r_{\rho j\beta} \Delta \bar{u}_{j\beta})
\]
Methodology

Transition from Damage Accumulation to Fatigue Cracking

★ Trigger fatigue cracking with plastic damage metrics
  ★ Fatigue cracks inserted in model (Cornell)

★ Metrics for damage accumulation
  ★ Total accumulated plastic slip over all slip systems
  ★ Maximum accumulated plastic slip on a single slip system
  ★ Maximum accumulated plastic slip on a slip plane
Implementation

- Software developed in C++
- Developed within Cornell FEM framework
- Utilized MPICH for parallel processing
- PETSc used for solving global system of equations
Implementation

★ Running on large-scale systems
★ SCOREC Medusa RedHat Linux cluster
★ 32 nodes, dual Xeon 2.0 GHz processors, 2 GB RAM
★ Cornell Theory Center Windows cluster
★ 170 nodes, dual Xeon 3.6 GHz processors, 4 GB RAM
★ SDSC DataStar AIX Unix system (p655 partition)
★ 272 nodes, eight 1.5 GHz processors, 16 GB RAM

★ Performance analysis [Scott Owens]

![Graphs showing run time vs. number of processors and run time vs. model size.](image-url)
Calibration Results

Calibrated Material Parameters

- $m = 0.005$
- $\dot{\gamma}_o = 1 \text{ s}^{-1}$
- $g_o = 220 \text{ MPa}$
- $\mu = 28.3 \text{ GPa}$
- $g_s = 250 \text{ MPa}$
- $\lambda = 60.9 \text{ GPa}$
- $G_o = 120 \text{ MPa}$
- $\eta = 5.1 \text{ GPa}$
Polycrystal with Embedded Particle

**Orientation Set #1**

**Orientation Set #2**

Load History

![Graph showing load history with strain on the y-axis and time (sec.) on the x-axis.](image)
Results

**TOTAL ACCUMULATED SLIP, TIME = 2.0 SEC.**

![Graph showing total accumulated slip over time for different orientations.](image)
Results

**TOTAL ACCUMULATED SLIP, TIME = 4.0 SEC.**

Total Accumulated Slip

- Orientation #1
- Orientation #2

Slip vs. Time graph
Results

**TOTAL ACCUMULATED SLIP, TIME = 6.0 SEC.**

![Graph showing total accumulated slip over time](image1)

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**Total Accumulated Slip**

![Graph showing total accumulated slip over time](image2)
Max Accumulated Slip on Single Slip System, Time = 2.0 sec.
Results

**Max Accumulated Slip on Single Slip System, Time = 4.0 sec.**

![Graph showing max accumulated slip over time for different orientations.](image)
Results

Max Accumulated Slip on Single Slip System, Time = 6.0 sec.
Max Accumulated Slip on Single Slip Plane, Time = 2.0 sec.
Results

Max Accumulated Slip on Single Slip Plane, Time = 4.0 sec.
Results

Max Accumulated Slip on Single Slip Plane, Time = 6.0 sec.
Future Work

★ Project results into high-cycle regime
  ★ Move between different time scales
★ Fatigue crack insertion
  ★ Dictated by plastic damage metrics
  ★ FEM model altered to include crack (Cornell)
★ Fatigue crack propagation
  ★ Direction of propagation influenced by microstructure
★ Particle-free zones
  ★ Material properties altered near grain boundaries
Questions?

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